#s 3, 4, 7, 8, 13

15TH

3 a) $S_{14}(3) = \sum_{n=0}^{14} \frac{x^n}{n!}$, where $x = 3$

sum(seq(1/(n!)*x^n, n, 0, 14, 1))

$S_{14}(3) \approx 20.08552346...$

b) $R_{14}(3) = \frac{f^{(15)}(c)}{15!} \cdot 3^{15}$

SIDEWORK: RECALL WE CAN USE MAX OF $f^{(15)}(c)$ AS A BOUND

$M = f^{(15)}(c)$ ← ALSO, C IS BETWEEN X & a SO (0 & 3)

$M = e^3$

$|R_{14}(3)| \le \frac{M}{15!} \cdot 3^{15}$

$|R_{14}(3)| \le \frac{e^3 \cdot 3^{15}}{15!} = .0002203954657$

$|R_{14}(3)| \le .00022...$

SO THIS MEANS OUR LAGRANGE APPROX. ~~ERROR IS WITHIN THREE UNITS OF e^3~~ IN THE 4TH D.P.

c) $e^3 = 20.08553692...$

SO ACTUAL ERROR IS $e^3 - S_{14}(3)$

$20.08553692 - 20.08552346$

$= .00001346...$

NOW COMPARE PART b) & c)

0.00022... VS. 0.00001346

LAGRANGE VS ACTUAL

SINCE LAGRANGE IS BIGGER... WE ARE OKAY.

#3 SIDEWORK FOR EXPANSION

$a = 0$

$f(x) = e^x$ $f(a) = 1$

$f'(x) = e^x$ $f'(a) = 1$

$f''(x) = e^x$ $f''(a) = 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \frac{x^{10}}{10!} + \dots \frac{x^n}{n!}$$

EIGHT TERMS

(4) $\ln(0.7) = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n} \cdot (x-1)^n$

a) $S_8(.7)$ = sum(seq(, n, 1, 8, 1) = $-0.3566[illegible]1944\ldots$

b) $R_8(.7) = \frac{f^9(c)}{9!} (-1)^9 (-.3)^9$ SIDEWORK:

smaller denom = MAX

$M = f^9(c)$ @ EITHER 1 OR .7

$|M| = f^9(.7) = \left|\frac{-8!}{x^9}\right| = \frac{8!}{(.7)^9}$

$|R_8(.7)| \leq \frac{M}{9!} (-.3)^9$

$|R_8(.7)| \leq \frac{-8!}{(.7)^9} \cdot \frac{(-.3)^9}{9!} = \frac{-8!}{9!} \cdot \left(\frac{-.3}{.7}\right)^9 = \frac{8!}{9!} \left(\frac{3}{7}\right)^9 \quad \frac{1}{9}\left(\frac{3}{7}\right)^9$

LAGRANGE ERROR. $|R_8(.7)| \leq \underbrace{0.0000}5419589\ldots$

SO IN THE [illegible] WE ARE WITHIN 2 UNITS.

c) $\ln(0.7) = -0.3566749439$

ACTUAL: $|\ln(0.7) - S_8(.7)| = .0000029993873\ldots$

THIS ✓s AS LAGRANGE ERROR IS BIGGER.

(7) ln(0.6) to seven DPS. TAYLOR → x = 1 (OR .6)

$|R_n(0.6)| \leq \frac{M}{(n+1)!} \cdot |(.6-1)^{n+1}|$; $M = f^{(n+1)}(c)$

$M = \frac{n!}{x^{n+1}} = \frac{n!}{.6^{n+1}}$

$\leq \frac{1}{n+1}\left(\frac{.4}{.6}\right)^{n+1}$

FOR SEVEN-PLACE ACCURACY.

$|R_n(0.6)| \leq \underbrace{\frac{1}{n+1}\left(\frac{2}{3}\right)^{n+1}}_{Y_1} < \underbrace{.00000005}$ 5.0×10^{-8}

ACCOUNTS FOR ROUNDING.

SCREW THE SOLVER :(

LOOK @ Y VALUES

WE'LL USE A TABLE

n = 1	Y = .2222
n = [illegible]	Y = .00458
n = 31	Y = 7.2×10^{-8}
n = 32	Y = 4.7×10^{-8}

so this occurs between 31 & 32 but closer to 32

$\boxed{\therefore n = 32}$

#4

SIDEWORK FOR EXPANSION:

$x = .7$

$a = 1$

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$ $\qquad f(a) = 0$

$f''(x) = -\frac{1}{x^2} = -x^{2}$ $\qquad f'(a) = 1$

$f'''(x) = \frac{2}{x^3} = 2x^{3}$ $\qquad f''(a) = -1$

$f^{4}(x) = -\frac{6}{x^4} = 3!$ $\qquad f'''(a) = -2$

$f^{4}(a) = -6$

$f^{5}(x) = -\frac{24}{x^5} = 4!$

$\vdots$

$f^{6}(x) = -\frac{120}{x^6} = 5!$

$f^{7}(x) = -\frac{720}{x^7} = 6!$

$f^{8}(x) = -\frac{5040}{x^8} = 7!$ $\qquad \frac{n!}{x^{n+1}}$

$\ln = 0 + 1(.7-1)^1 - \frac{1}{2!}(-.3)^2 - \frac{2}{3!}(-.3)^3 - \frac{6}{4!}(-.3)^3$

$= 0 + \frac{1}{1}(-.3)^1 - \frac{1}{2}(-.3)^2 + \frac{1}{3}(-.3)^3 - \frac{1}{4}(-.3)^4 + \ldots$

$= 0 - .3 - .045 - .009 - .002025 - .000486 \ldots$

$= -0.35667494$

8) $e^{(10)}$ to 5 DPs MACLAURIN

FROM #3

$|R_n(?)| \le \frac{M}{(n+1)!}(x-a)^{n+1}$ → SO HERE $e^{10} = M$

& $M = f^{(n+1)}(c)$

$|R_n(10)| \le \frac{e^{10}}{(n+1)!}(10)^{n+1} \le .000005$ or 5.0×10^{-6}

$\boxed{\text{therefore } x = 10}$

AGAIN, USE TABLE NOT SOLVER !!

x	Y_1
0	220265
10	1.10×10^6
23	94200.4
35	.059...
41	1.56×10^{-5}
42	3.46×10^{-6}

SO IT'S BETWEEN 41 & 42 BUT CLOSER TO 42

SO $n = 42$

(REMEMBER FOR e^x WE START @ 0
SO WE'D NEED 43 TERMS)

12-8

4

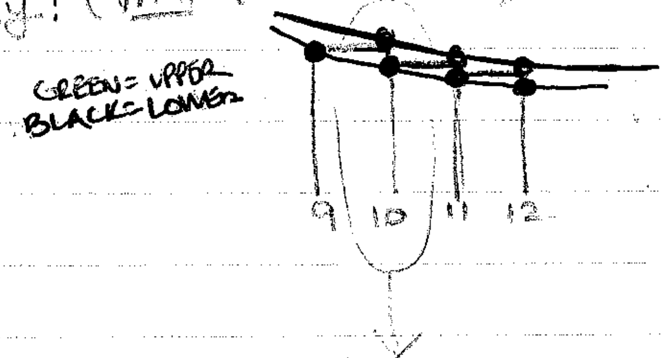
13) $S=\sum_{n=1}^{\infty}\frac{1}{n^3}$

a) S_{10} = sum(seq(1/(n^3),n,1,10,1))

$S_{10}=1.197531986...$

SO WE KNOW UPPER & LOWER BOUNDS HAPPEN BETWEEN 10 & 11

VISUALLY: (WAY ZOOMED IN)



SO LETS FIND EACH ONE

UPPER (R_{10}) $< \int_{10}^{b}\frac{1}{x^3}\,dx \xRightarrow{\text{FROM } 9,10} \lim_{b\to\infty}\left(-\frac{1}{2x^2}\Big|_{10}^{b}\right)=$

$-\frac{1}{200}-\left(-\frac{1}{2\cdot\infty}\right)=-\frac{1}{200}$

LOWER (R_{11}) $> \int_{11}^{b}\frac{1}{x^3}\,dx \Rightarrow \lim_{b\to\infty}\left(-\frac{1}{2x^2}\Big|_{11}^{b}\right)=-\frac{1}{242}$

SO R_{10} AVEROUND $=\dfrac{\left(-\frac{1}{200}+\left(-\frac{1}{242}\right)\right)}{2}=0.004566116$

total S = S_{10} + R_{10} AVEROUND

$=1.197531986+0.004566116$

$=1.20209810...$

13a) CONT.

MAX ERROR = AVE DISTANCE BETWEEN THE UPPER & LOWER BOUNDS

$$< \left| \frac{R_{10} - R_{11}}{2} \right|$$

$$< \left| \frac{-\frac{1}{200} + \frac{1}{242}}{2} \right|$$

$$< .000433884297\ldots$$

SO ≈ 3 DPS OF ACCURACY.

b) $R_n < .000005$ OR 5.0×10^{-6}

R_n = ACCURACY IS BIGGER THAN $\underline{10}$. (FROM ABOVE ONLY 3 DPS)

$\therefore$ LETS USE MAX ERROR IDEA WITH n, & $n+1$ INSTEAD OF 10 & 11

$$\text{MAX ERROR} < \left| \frac{R_{n+1} - R_n}{2} \right|$$

$$\text{M.E.} < \left| \frac{1}{2}\int_{n+1}^{b} x^{-3} - \frac{1}{2}\int_{n}^{b} x^{-3} \right|$$

$$\text{ME} < \left| \underbrace{-.25(n+1)^{-2} + .25n^{-2}}_{Y_1} \right| \underbrace{< 5.0 \times 10^{-6}}_{\text{USE TABLE}}$$

45	5.3×10^{-6}
46	4.97×10^{-6}

SO $n = 46$

[SO 46 TERMS]

13b) CONT.

FOR ONLY UPPER, WE HAVE

MAX ERROR $< |R_{1n}| <$ .000005 or 5.0×10^{-6}

$$\left| \int_n^b \frac{1}{x^3} \right| < .000005$$

$$\left| -\frac{1}{2} n^{-2} \right| < .000005$$

$$n^{-2} = .00001$$

$$n^2 = \frac{1}{.00001}$$

$$n = \sqrt{\frac{1}{.00001}}$$

$$n = 316.2277660$$

so between 316 & 317!!! term!