2.1 Functions and Their Graphs

Goals
- Represent relations and functions.
- Graph and evaluate linear functions.

Your Notes

VOCABULARY

Relation A pairing of input values with output values

Domain of a relation The set of input values for a relation

Range of a relation The set of output values for a relation

Function A relation with exactly one output for each input

Ordered pair A pair of numbers of the form (x, y) that represents a point in the coordinate plane

Coordinate plane A plane divided into four quadrants by the x-axis and the y-axis that is used to plot ordered pairs

Equation in two variables An equation such as $y = 2x + 1$

Solution of an equation in two variables An ordered pair (x, y) that makes the equation a true statement when the values of x and y are substituted into the equation

Independent variable The input variable in an equation

Dependent variable The output variable in an equation

Graph of an equation in two variables The collection of all points (x, y) whose coordinates are solutions of the equation

Linear function A function of the form $y = mx + b$ where m and b are constants

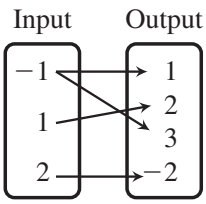
Function notation Use of the symbol $f(x)$ for the dependent variable of a function

Your Notes

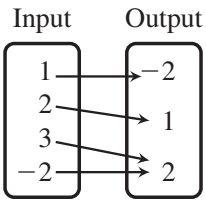
Example 1 *Identifying Functions*

Identify the domain and range. Is the relation a function?

a.



b.



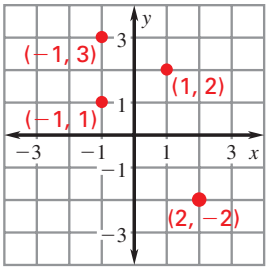
a. The domain consists of −1, 1, and 2, and the range consists of 1, 2, 3, and −2. The relation is not a function because the input −1 is mapped onto both 1 and 3.

b. The domain consists of 1, 2, 3, and −2, and the range consists of −2, 1, and 2. The relation is a function because each input is mapped to exactly one output.

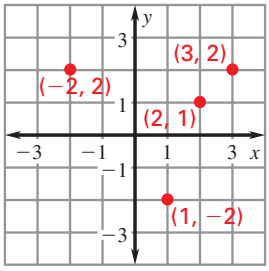
Example 2 *Graphing Relations*

Graph the relations given in Example 1.

a. Write the relation as a set of ordered pairs: (−1, 1), (−1, 3), (1, 2), (2, −2). Plot the points.



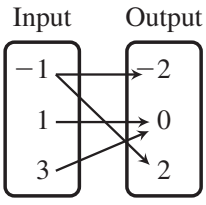
b. Write the relation as a set of ordered pairs: (1, −2), (2, 1), (3, 2), (−2, 2). Plot the points.



✔ ***Checkpoint*** **Complete the following exercise.**

1. Identify the domain and range and tell whether the relation is a function. Write the relation as a set of ordered pairs.

domain: −1, 1, 3; range: −2, 0, 2; not a function; (−1, −2), (−1, 2), (1, 0), (3, 0)



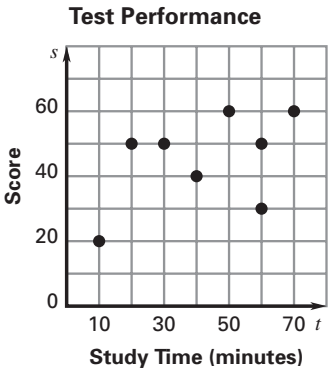
Your Notes

VERTICAL LINE TEST FOR FUNCTIONS

A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

Example 3 *Using the Vertical Line Test in Real Life*

The graph shows the scores s and time spent studying t for several students who took a test. Are the scores a function of the time spent studying? Explain.



Solution

The scores are not a function of the time spent studying because there is a vertical line that intersects the graph at more than one point. A vertical line passes through both (60 , 30) and (60 , 50).

GRAPHING EQUATIONS IN TWO VARIABLES

To graph an equation in two variables, follow these steps.

Step 1 Construct a table of values .

Step 2 Graph enough solutions to recognize a pattern .

Step 3 Connect the points with a line or a curve .

Example 4 *Graphing a Function*

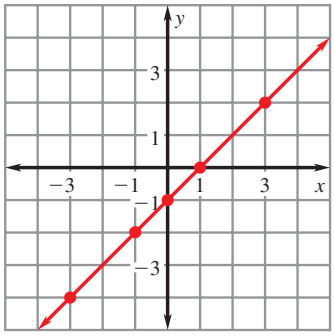
Graph the function $y = x - 1$.

1. Make a table of values.

Choose x.	−3	−1	0	1	3
Evaluate y.	−4	−2	−1	0	2

2. Plot the points. Notice the five points lie on a line .

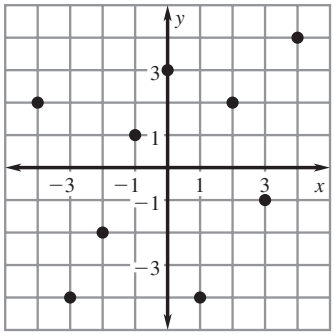
3. Draw a line through the points.



Your Notes

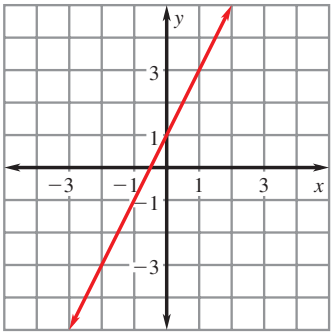
✔ ***Checkpoint*** **Complete the following exercises.**

2. Use the graph to determine if the plotted points represent a function.



function

3. Graph the function $y = 2x + 1$.



Example 5 ***Evaluating Functions***

Evaluate the function when $x = 2$.

a. $f(x) = -3x + 4$ **b.** $f(x) = x^2 - 4x - 1$

Solution

a. $f(x) = -3x + 4$ **Write function.**

$f(\underline{2}) = -3(\underline{2}) + 4$ **Substitute for *x*.**

$= \underline{-2}$ **Simplify.**

b. $f(x) = x^2 - 4x - 1$ **Write function.**

$f(\underline{2}) = \underline{2}^2 - 4(\underline{2}) - 1$ **Substitute for *x*.**

$= \underline{-5}$ **Simplify.**

✔ ***Checkpoint*** **Evaluate the function for $x = -1$.**

4. $f(x) = 3x^2 + 2$

5

5. $f(x) = -x + 9$

10

Homework