Can a Human Break the Sound Barrier?

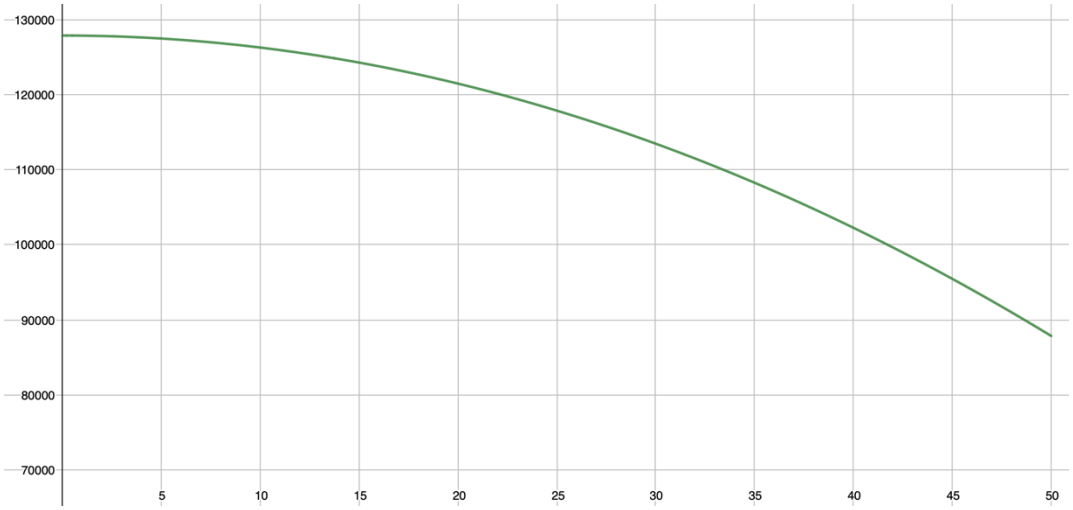
Name:_____

On October 14th, 2012, Austrian skydiver Felix Baumgartner broke a world record for a high-altitude dive when he ascended 127,850 feet in a helium balloon and then went into a free fall lasting more than 4 minutes.

1. Baumgartner is in free fall for 4 minutes and 20 seconds (260 seconds) before he deploys his parachute at an elevation of 8,420 feet above sea level.
   a. What was the vertical distance of the freefall?

   b. What was his average velocity during the freefall?

2. His elevation (in feet) above sea-level, t seconds after stepping off the balloon can be approximated by $f(t) = 127850 - 16t^2$ for $0 \leq t \leq 50$.
   a. Look at the graph of $f(t)$ below. Label both axes.



   b. Was Baumgartner traveling at a constant velocity? How do you know?

   c. What time does it look like Baumgartner is traveling the fastest? How can you tell?

3. Let's see if we can estimate his velocity exactly 30 seconds after leaving the balloon.
   a. What is his average velocity between $t = 20$ and $t = 30$? Show your work.

   Is this faster or slower than the velocity at exactly 30 seconds? Explain.

   b. What is his average velocity between $t = 30$ and $t = 40$? Show your work.

   Is this faster or slower than the velocity at exactly 30 seconds? Explain.

4. Let's take an interval even closer to 30.
    a. Find the average velocity between $t = 29$ and $t = 30$. Show your work.

    b. Find the average velocity between $t = 30$ and $t = 31$. Show your work.

5. Are the estimates in 4a and 4b better or worse than those in 3a and 3b? Why?

6. How could we get an even better estimate?

7. We're going to find the average velocity between $t = 30$ and $t = 30 + h$. Let's break it down into steps.
    a. Find $f(30 + h)$. Simplify.

    b. Find $f(30 + h) - f(30)$.

    c. Write the expression for $\frac{f(30+h)-f(30)}{h}$ using what you found above.

    d. What value of *h* would represent his velocity at *exactly* $t = 30$? Explain.

    e. Show how you could determine this velocity.

8. The speed of sound is 1,125.3 feet per second. Did Baumgartner go supersonic?

Topic 2.1—Instantaneous Rates of Change

Important Ideas:

Check Your Understanding!

1. Let $f(x) = x^2 - 4x$.
   a. Find the average rate of change on the interval [-1,5].

   b. Find the instantaneous rate of change at $x = 3$.

2. Write, but do not evaluate, an expression that gives the instantaneous rate of change of $g(x) = \frac{-1}{3x}$ at $x = 2$.

3. Which of the following gives the instantaneous rate of change of $f(x)$ at $x = -1$. Choose all that apply:

$\lim_{h\to\infty} \frac{f(-1+h)-f(-1)}{h}$ $\lim_{h\to 0} \frac{f(-1+h)-f(-1)}{h}$ $\lim_{h\to -1} \frac{f(-1+h)-f(-1)}{h}$

$\lim_{x\to a} \frac{f(x)-f(-1)}{-1}$ $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $\lim_{x\to -1} \frac{f(-1)-f(a)}{x-(-1)}$

$\lim_{x\to -1} \frac{f(x)-f(-1)}{x-1}$ $\lim_{x\to -1} \frac{f(x)-f(-1)}{x-(-1)}$