Can a Human Break the Sound Barrier? Name: Key

On October 14th, 2012, Austrian skydiver Felix Baumgartner broke a world record for a high-altitude dive when he ascended 127,850 feet in a helium balloon and then went into a free fall lasting more than 4 minutes.

1. Baumgartner is in free fall for 4 minutes and 20 seconds (260 seconds) before he deploys his parachute at an elevation of 8,420 feet above sea level.
   a. What was the vertical distance of the freefall?

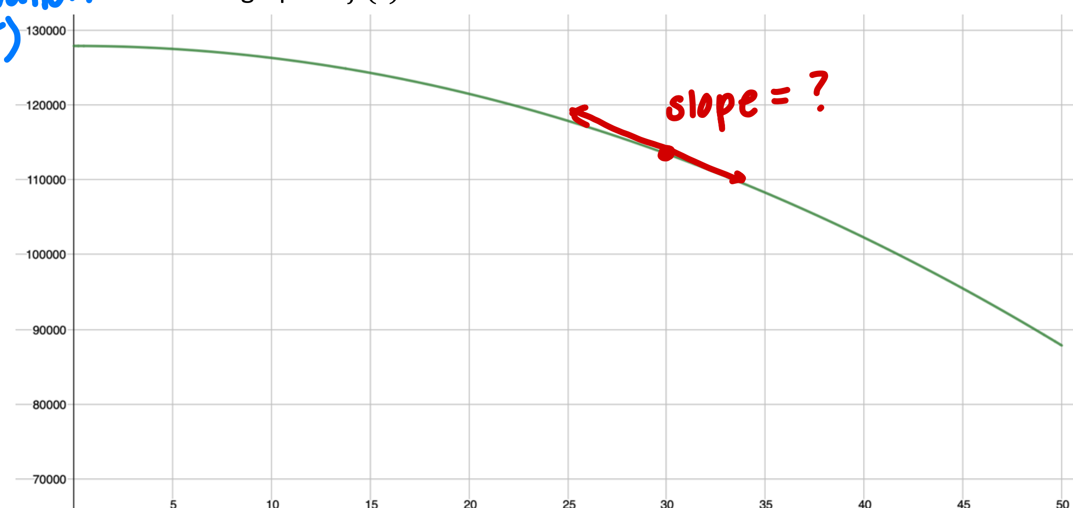
   127,850 − 8420 = 119,430 feet in free fall

   b. What was his average velocity during the freefall?

   $$\frac{-119{,}430 \text{ ft}}{260 \text{ s}} = -459.346 \text{ ft/s}$$

2. His elevation (in feet) above sea-level, t seconds after stepping off the balloon can be approximated by $f(t) = 127850 - 16t^2$ for $0 \le t \le 50$.
   a. Look at the graph of $f(t)$ below. Label both axes.

Elevation (feet)



time (seconds)

   b. Was Baumgartner traveling at a constant velocity? How do you know? speed w/ direction

   No, there's not a constant slope.

   c. What time does it look like Baumgartner is traveling the fastest? How can you tell?

   Near the end around t = 50 because slope is steepest

3. Let's see if we can estimate his velocity exactly 30 seconds after leaving the balloon.
   a. What is his average velocity between $t = 20$ and $t = 30$? Show your work.

   $$\frac{f(30-10)-f(30)}{-10}$$

   $$\frac{f(30)-f(20)}{30-20} = \frac{113450-121450}{10} = -800 \text{ ft/s}$$

   Is this faster or slower than the velocity at exactly 30 seconds? Explain.

   Slower because the slope between t=20 and t=30 is flatter than the slope right at t=30.

   b. What is his average velocity between $t = 30$ and $t = 40$? Show your work.

   $$\frac{f(30+10)-f(30)}{10}$$

   $$\frac{f(40)-f(30)}{40-30} = \frac{10{,}2250-113{,}450}{10} = -1{,}120 \text{ ft/s}$$

   Is this faster or slower than the velocity at exactly 30 seconds? Explain.

   faster, the slope gets steeper after t=30.

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4. Let's take an interval even closer to 30.

$\frac{f(30-1)-f(30)}{-1}$

   a. Find the average velocity between $t = 29$ and $t = 30$. Show your work.

$$\frac{f(30)-f(29)}{30-29} = \frac{113450-114394}{1} = -944 \text{ ft/s}$$

$\frac{f(30+1)-f(30)}{1}$

   b. Find the average velocity between $t = 30$ and $t = 31$. Show your work.

$$\frac{f(31)-f(30)}{31-30} = \frac{112474-113450}{1} = -976 \text{ ft/s}$$

5. Are the estimates in 4a and 4b better or worse than those in 3a and 3b? Why?

Better! They're closer together so we're narrowing in on the true slope of the curve at $t=30$.

6. How could we get an even better estimate?

Make the interval even smaller!

STOP

7. We're going to find the average velocity between $t = 30$ and $t = 30 + h$. Let's break it down into steps.

   a. Find $f(30 + h)$. Simplify.

$$127850-16(30+h)^2 = 127850-16(900+60h+h^2)$$
$$= 113450-960h-16h^2$$

   b. Find $f(30 + h) - f(30)$.

$$113450-960h-16h^2-113450 = -960h-16h^2$$

Difference Quotient

$\frac{f(30+h)-f(30)}{h}$

   c. Write the expression for $\frac{f(30+h)-f(30)}{h}$ using what you found above.

$$\frac{-960h-16h^2}{h}$$

   d. What value of h would represent his velocity at *exactly* $t = 30$? Explain.

We need h to be 0 so no time has elapsed.

Taking $\lim_{h\to 0}$ gives instantaneous rate of change

   e. Show how you could determine this velocity.

Since we can't divide by 0, we need to let h approach 0.

$$\lim_{h\to 0}\frac{-960h-16h^2}{h} = \lim_{h\to 0}\left(\frac{h(-960-16h)}{h}\right) = \lim_{h\to 0}(-960-16h)$$
$$= -960 \text{ ft/s}$$

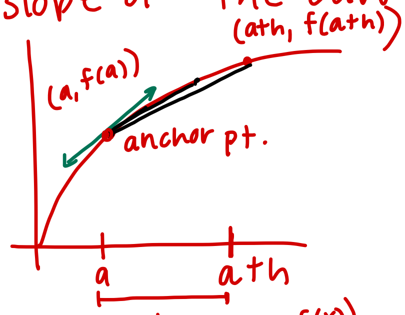
8. The speed of sound is 1,125.3 feet per second. Did Baumgartner go supersonic?

No, 960 ft/s < 1,125.3 ft. per second.

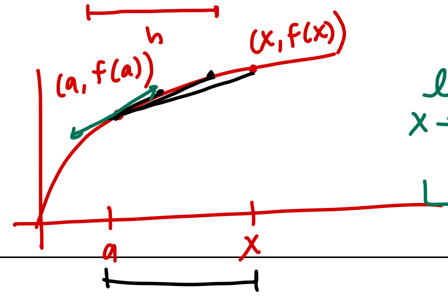
Topic 2.1—Instantaneous Rates of Change

Important Ideas:

Instantaneous rate of change at $x=a$ represents the slope of the curve at $x=a$.



$$\underbrace{\lim_{h\to 0} \underbrace{\frac{f(a+h)-f(a)}{h}}_{\text{avg. R.O.C}}}_{\text{instantaneous R.O.C.}}$$ Definition 1



$$\underbrace{\lim_{x\to a} \underbrace{\frac{f(x)-f(a)}{x-a}}_{\text{avg. R.O.C.}}}_{\text{instantaneous R.O.C.}}$$ Definition 2

Check Your Understanding!

1. Let $f(x) = x^2 - 4x$.
   a. Find the average rate of change on the interval [-1,5].

   $$\frac{f(5)-f(-1)}{5-(-1)} = \frac{5-5}{6} = 0$$

   b. Find the instantaneous rate of change at $x = 3$.

   $$\lim_{h\to0}\frac{(3+h)^2-4(3+h)-(-3)}{h} = \lim_{h\to0}\frac{9+6h+h^2-12-4h+3}{h}$$
   $$= \lim_{h\to0}\frac{2h+h^2}{h} = \lim_{h\to0}\frac{h(2+h)}{h} = \lim_{h\to0}(2+h) = 2$$

2. Write, but do not evaluate, an expression that gives the instantaneous rate of change of $g(x) = \frac{-1}{3x}$ at $x = 2$.

   $$\lim_{h\to0}\frac{g(2+h)-g(2)}{h} = \lim_{h\to0}\frac{\frac{-1}{3(2+h)}-\frac{-1}{6}}{h} \quad \text{or} \quad \lim_{x\to2}\frac{\frac{-1}{3x}-\left(\frac{-1}{6}\right)}{x-2}$$

3. Which of the following gives the instantaneous rate of change of $f(x)$ at $x = -1$. Choose all that apply:

$\lim_{h\to\infty}\frac{f(-1+h)-f(-1)}{h}$ $\quad$ $\lim_{h\to0}\frac{f(-1+h)-f(-1)}{h}$ $\quad$ $\lim_{h\to-1}\frac{f(-1+h)-f(-1)}{h}$

$\lim_{x\to a}\frac{f(x)-f(-1)}{-1}$ $\quad$ $\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$ $\quad$ $\lim_{x\to-1}\frac{f(-1)-f(a)}{x-(-1)}$

$\lim_{x\to-1}\frac{f(x)-f(-1)}{x-1}$ $\quad$ $\lim_{x\to-1}\frac{f(x)-f(-1)}{x-(-1)}$