Name______________________________

Calc BC – Notes Section 5-3

$$\int f(x)dx$$

WITH RESPECT TO X

INTEGRAND

INTEGRAL SIGN

THIS IS AN OPERATOR IT MEANS EVALUATE ME! (OR INTEGRATE ME)

Indefinite Integral: (AKA: ANTI DERIVATIVE)

$$g(x) = \int f(x)dx \iff g'(x) = f(x) + C$$

Integral of a constant times a function

$$\int 3f(x)dx = 3\int f(x)dx$$

Integral of a Sum of two functions

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

Integral of the power function

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C}$$

ex) $\int 3x^2 dx$

$= x^3 + C$

ex) $\int 12x^2 dx$

$= 4x^3 + C$

Proof:

$$\int b^x\,dx = \int \left(e^{\ln b}\right)^x dx$$

let $u = \ln b x$

$du = \ln b\, dx$

$\frac{du}{\ln b} = dx$

$$\frac{1}{\ln b}\int e^u\,du$$

$$\frac{1}{\ln b}e^u + c$$

$$\frac{1}{\ln b}e^{\ln bx} + c$$

$$\boxed{= \frac{1}{\ln b}b^x + c}$$

Q.E.D.

Integral of the natural exponent function

$$\int e^x\,dx = e^x + C$$

Derivative and Integral of Base-b Exponential functions

RECALL:

$y = b^x$

$y' = \ln b \cdot b^x \cdot (x')$

$$\int y = \frac{b^x}{\ln b} + C$$

Finding an integral using the method of substitution (u subst)

Step 1: Make a choice for u, say ________________

Step 2: Compute ____________________

Step 3: Make the substitution __________________________

** At this stage, the entire integral must be in terms of u; no x's should remain. If this is not the case, try a different choice of u. even your a & b values

Step 4: Evaluate the resulting integral, if possible.

Step 5: Replace ________________, so that the final answer is in terms of x. $\int_a^b$

For the integral

$$\int e^{\cos x}\sin x\,dx$$

Let u = $\cos x$ then du = $-\sin x\,dx$

$$\frac{du}{-\sin x} = dx$$

Then solve the following integral:

$$\int e^u \sin x \cdot \frac{du}{-\sin x} = \int -e^u\,du = -\int e^u\,du$$

$$= -e^u + C$$

Replace your u value, and get the final answer of

$$\boxed{= -e^{\cos x} + C}$$

a.) $\int \frac{dx}{\left(\frac{1}{3}-x\right)^5}$ let $u = \frac{1}{3} - x$

then $du = -1\,dx$

$-du = dx$

$\int -u^{-5}\,du = \frac{-1}{-4} u^{-4} + c$

$\boxed{= \frac{1}{4\left(\frac{1}{3}-x\right)^4} + c}$

b.) $\int e^x\left(3-4e^x\right)dx$

let $u = 3 - 4e^x$

then $du = -4e^x\,dx$

$\frac{du}{-4e^x} = dx$

$\int e^x(u)\frac{du}{-4e^x}$

$= -\frac{1}{4}\int u\,du$

$= -\frac{1}{4}\frac{u^2}{2} + c$

$\boxed{= -\frac{(3-4e^x)^2}{8} + c}$

**** You should always check your answers by taking the derivative of your integral. It should be what you started with!!! SHOW FOR ONE PROBLEM!

Examples

1.) $\int (3x+7)^2\,dx$ let $u = 3x+7$

then $du = 3dx$

$\frac{du}{3} = dx$

$\int u^2 \frac{du}{3}$

$\frac{1}{3}\int u^2 du = \frac{1}{3}\frac{u^3}{3} + c = \boxed{\frac{(3x+7)^3}{9} + c}$

2.) $\int \cos 8x\,dx$

$u = 8x$

$du = 8\,dx$

$\frac{du}{8} = dx$

$\frac{1}{8}\int \cos u\,du$

$\boxed{\frac{1}{8}\cos 8x + c}$

3.) $\int \csc^2(\pi x)\,dx$ let $u = \pi x$

$du = \pi\,dx$

$\frac{du}{\pi} = dx$

$\frac{1}{\pi}\int \csc^2 u\,du$

$-\frac{1}{\pi}\cot u + c$ $\boxed{\frac{-1}{\pi}\cot(\pi x) + c}$

4.) $\int \cos^3 x \sin x\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$\frac{du}{-\sin x} = dx$

$-\int u^3\,du$

$-\frac{u^4}{4} + c$ $\boxed{-\frac{\cos^4 x}{4} + c}$

5.) $\int 3^x\,dx$

let $u = x$
$du = 1dx$

$$\boxed{\frac{3^x}{\ln 3} + C}$$

6.) $\int e^{5x}\,dx$

let $u = 5x$
$du = 5dx$
$\frac{du}{5} = dx$

$\frac{1}{5}\int e^u\,du$

$$\boxed{\frac{1}{5}e^{5x} + C}$$

WOO HOO... NOW LETS GO BACKWARDS!

For the following examples, find an equation for the indefinite integral. Why is the word "an" used instead of "the"? BECAUSE ∃ ∞ ANTIDERIVATIVES FOR ANY DERIVATIVE

6.) $dy = 8x^3 dx$

$\int dy = \int 2 \cdot 4x^3 dx$

LOOKS LIKE SOMETHING x^4 TO START

$y = 2x^4 + C$

WHERE C IS SOME CONSTANT

7.) $\int dy = \int (3x+2)^3 dx$

let $u = 3x+2$

$du = 3dx$

$y = \frac{u^4}{4} \frac{1}{3} du$

$\boxed{y = \frac{1}{12}(3x+2)^4 + C}$

$\int \frac{u^4}{4}$

8.) $dy = \cos 2x dx$

let $u = 2x$

$du = 2dx$

$\frac{1}{2} du = dx$

$y = \sin(2x) \frac{1}{2} du$

$\boxed{y = \frac{1}{2}\sin(2x) + C}$

9.) $dy = e^{\sin x} \cos x \, dx$

let $u = \sin x$

$du = \cos x \, dx$

$y = \int e^u du$

$\boxed{y = e^{\sin x} + C}$

10.) $dy = \tan x \sec^2 x \, dx$

let $u = \tan x$

$du = \sec^2(x) dx$

$y = \int u \, du$

$y = \frac{u^2}{2} + C$

$\boxed{y = \frac{\tan^2(x)}{2} + C}$

Find both dy and Δy for the given values of x and Δx

11.) $y = \sqrt{x^3 + 1}$ x = 3, dx = 0.1

$dy = \frac{1}{2}(x^3+1)^{-\frac{1}{2}} \cdot 3x^2 \cdot dx$

$dy = \frac{3x^2}{2(x^3+1)^{\frac{1}{2}}} dx$

NOTE: $dy \neq \Delta y$

change in y on tangent $\rightarrow dy = \frac{3(3)^2}{2(3^3+1)^{\frac{1}{2}}}(0.1) \approx .255126$

change in y on the curve (f(x)) $\rightarrow \Delta y = y(3.1) - y(3)$

$= \sqrt{(3.1)^3+1} - \sqrt{(3)^3+1} \approx .25746$