10.2 Arcs and Chords

Goals
- Use properties of arcs of circles.
- Use properties of chords of circles.

VOCABULARY

Central angle An angle whose vertex is the center of a circle

Minor arc Part of a circle that measures less than 180°

Major arc Part of a circle that measures between 180° and 360°

Semicircle An arc whose endpoints are the endpoints of a diameter of the circle

Measure of a minor arc The measure of its central angle

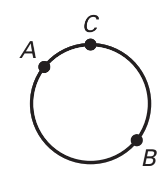
Measure of a major arc The difference between 360° and the measure of its associated minor arc

Congruent arcs Two arcs of the same circle or of congruent circles that have the same measure

POSTULATE 26: ARC ADDITION POSTULATE

The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

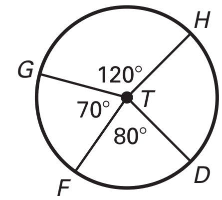
$m\overset{\frown}{ABC} = \underline{m\overset{\frown}{AB}} + \underline{m\overset{\frown}{BC}}$



Example 1 *Finding Measures of Arcs*

Find the measure of each arc.

a. $\overset{\frown}{DG}$ **b.** $\overset{\frown}{DGH}$ **c.** $\overset{\frown}{DH}$



Solution

a. $m\overset{\frown}{DG} = m\overset{\frown}{DF} + m\overset{\frown}{FG} = \underline{80}° + \underline{70}° = \underline{150}°$

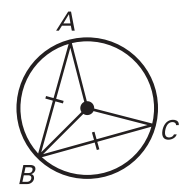
b. $m\overset{\frown}{DGH} = m\overset{\frown}{DG} + m\overset{\frown}{GH} = \underline{150}° + \underline{120}° = \underline{270}°$

c. $m\overset{\frown}{DH} = 360° - m\overset{\frown}{DGH} = 360° - \underline{270}° = \underline{90}°$

THEOREM 10.4

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

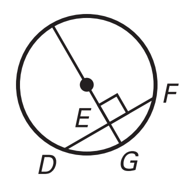
$\overset{\frown}{AB} \cong \overset{\frown}{BC}$ if and only if $\underline{\overline{AB}} \cong \underline{\overline{BC}}$.



THEOREM 10.5

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

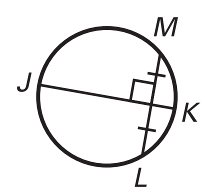
$\overline{DE} \cong \overline{EF}$, $\underline{\overset{\frown}{DG}} \cong \underline{\overset{\frown}{GF}}$



THEOREM 10.6

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.

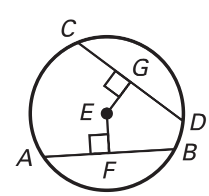
$\underline{\overline{JK}}$ is a diameter of the circle.



THEOREM 10.7

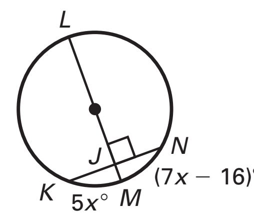
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

$\overline{AB} \cong \overline{CD}$ if and only if $\underline{\overline{EF}} \cong \underline{\overline{EG}}$.



Example 2 *Using Theorem 10.5*

Find $m\overset{\frown}{KM}$ using Theorem 10.5.

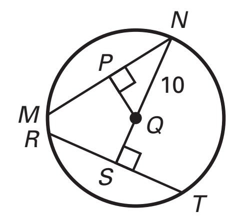


$m\overset{\frown}{KM} = m\overset{\frown}{MN}$		**Theorem 10.5**
$5x° = (7x - 16)°$		**Substitute.**
$\underline{0} = \underline{2}\,x - 16$		**Subtract $\underline{5x}$ from each side.**
$\underline{16} = \underline{2}\,x$		**Add $\underline{16}$ to each side.**
$\underline{8} = x$		**Divide.**

Answer $m\overset{\frown}{KM} = 5x° = \underline{(5 \cdot 8)}° = \underline{40}°.$

Example 3 *Using Theorem 10.7*

Find QS if $MN = 16$, $RT = 16$, and $NQ = 10$.



Because $\overline{MN}$ and $\overline{RT}$ are congruent chords, they are equidistant from the center. So, $\overline{PQ} \cong \overline{QS}$. To find QS, first find PN.

$\overline{PQ} \perp \overline{MN}$, so $\overline{PQ}$ bisects $\overline{MN}$. Because $MN = 16$,

$PN = \dfrac{16}{2} = \underline{8}$.

Then use PN to find PQ.

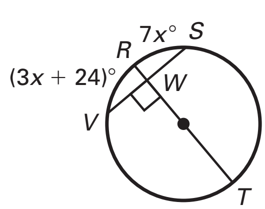
$PN = \underline{8}$ and $NQ = 10$, so $\triangle NPQ$ is a $\underline{6}$-$\underline{8}$-$\underline{10}$ right triangle. So, $PQ = \underline{6}$.

Finally, use PQ to find QS.

Answer Because $\overline{QS} \cong \overline{PQ}$, $QS = PQ = \underline{6}$.

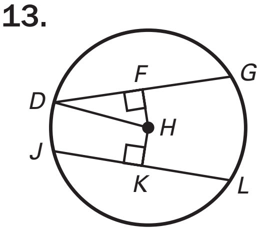
✔ ***Checkpoint*** **Complete the following exercises.**

1. Use Theorem 10.5 to find $m\overset{\frown}{RS}$.



42°

2. Find HK if $DG = JL = 24$, and $DH = 13$.



5