2.2 Definitions and Biconditional Statements

Goals
- Recognize and use definitions.
- Recognize and use biconditional statements.

VOCABULARY

Perpendicular lines Two lines are perpendicular lines if they intersect to form a right angle.

Line perpendicular to a plane A line perpendicular to a plane is a line that intersects the plane in a point and is perpendicular to every line in the plane that intersects it.

Biconditional statement A biconditional statement is a statement that contains the phrase "if and only if."

Example 1 | Using Definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.

a. $\angle KLJ$ and $\angle KJL$ are complementary.

b. $\overleftrightarrow{KL}$ and $\overleftrightarrow{LJ}$ are perpendicular.

c. $\angle MKJ$ is a right angle.
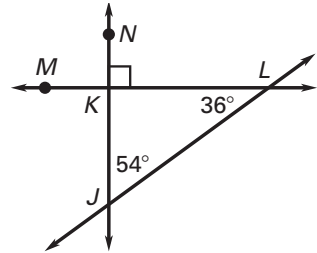
Solution

a. This statement is true . Two angles are complementary if the sum of their measures is 90° . $m\angle KLJ + m\angle KJL = $ 90° , so the angles are complementary .

b. This statement is false . $\overleftrightarrow{KL}$ and $\overleftrightarrow{LJ}$ do not intersect to form a right angle . So, the lines are not perpendicular .

c. This statement is true . $\angle MKJ$ and $\angle NKL$ are vertical angles. $\angle NKL$ is a right angle . Because vertical angles are congruent, $\angle MKJ$ is a right angle .

✔ *Checkpoint* **Use the diagram in Example 1 to decide whether the statement is true. Explain your answer using the definitions you have learned.**

1. ∠*KJL* is an acute angle.	**2.** Point *N* is in the interior of ∠*KLJ*.
True. An acute angle has a measure between 0° and 90°. The measure of ∠*KJL* is 54°, so the angle is acute.	False. A point is in the interior of an angle if it is between points that lie on each side of the angle. Point *N* is in the exterior of ∠*KLJ*.

Example 2 *Rewriting a Biconditional Statement*

Rewrite the following biconditional statement as a conditional statement and its converse.

An angle is a straight angle if and only if its measure is 180°.

Conditional statement: If an angle is a straight angle , then its measure is 180° .

Converse: If the measure of an angle is 180° , then the angle is a straight angle .

Example 3 *Analyzing a Biconditional Statement*

Consider the following statement: $x = 2$ **if and only if** $3x + 5x = 10x - 2x$.

a. Is this a biconditional statement? **b.** Is the statement true?

Solution

a. The statement is biconditional because it contains the phrase "if and only if" .

b. The statement can be rewritten as the following statement and its converse.

> Are there any values other than $x = 2$ that make the equation true?

  Conditional statement: If $x = 2$, then $3x + 5x = 10x - 2x$.
  Converse: If $3x + 5x = 10x - 2x$, then $x = 2$.

The first statement is true . The second statement is false . So, the biconditional statement is false .

Example 4   *Writing a Biconditional Statement*

Each of the following statements is true. Write the converse of each statement and decide whether the converse is *true* or *false*. If the converse is true, combine it with the original statement to form a true biconditional statement. If the converse is false, state a counterexample.
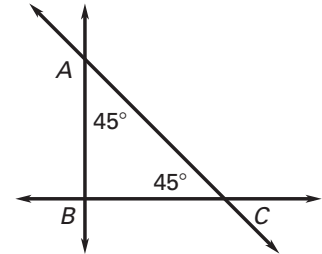
a. If $\sqrt{x} = 1$, then $x = 1$.

b. If two angles are vertical angles, then they are congruent.

Solution

a. Converse: <u>If $x = 1$, then $\sqrt{x} = 1$</u> . The converse is <u>true</u> .
   Biconditional statement: <u>$\sqrt{x} = 1$ if and only if $x = 1$.</u>

b. Converse: <u>If two angles are</u>
   <u>congruent, then they are vertical</u>
   <u>angles</u> . The converse is <u>false</u> . As a
   <u>counterexample</u> , consider the figure
   at the right. $\angle$ <u>BAC</u> and $\angle$ <u>BCA</u> are
   congruent, but they are not <u>vertical</u>
   <u>angles</u> .

[Figure: two lines crossing through points A, B, C with angles of 45° marked]

✔ *Checkpoint* **Complete the following exercises.**

3. Rewrite the following biconditional statement as a conditional statement and its converse.

Two angles are supplementary if and only if the sum of their measures is 180°.

Conditional statement: If two angles are supplementary, then the sum of their measures is 180°.

Converse: If the sum of the measures of two angles is 180°, then the angles are supplementary.

4. Consider the following statement: Two segments are congruent if and only if they have the same length.

   a. Is the statement biconditional?   yes

   b. Is the statement *true* or *false*?   true