2.1 Conditional Statements

Goals
- Recognize and analyze a conditional statement.
- Write postulates about points, lines, and planes using conditional statements.

VOCABULARY

Conditional statement A conditional statement is a type of logical statement that has two parts, a hypothesis and a conclusion.

If-then form If-then form of a conditional statement uses the words "if" and "then." The "if" part contains the hypothesis and the "then" part contains the conclusion.

Hypothesis A hypothesis is the "if" part of a conditional statement.

Conclusion A conclusion is the "then" part of a conditional statement.

Converse The converse of a conditional statement is formed by switching the hypothesis and the conclusion.

Negation The negation of a statement is formed by writing the negative of the statement.

Inverse An inverse is the statement formed when you negate the hypothesis and conclusion of a conditional statement.

Contrapositive A contrapositive is the statement formed when you negate the hypothesis and conclusion of the converse of a conditional statement.

Equivalent statements When two statements are both true or both false, they are called equivalent statements.

Example 1 ***Rewriting in If-Then Form***

Rewrite the conditional statement in *if-then* form.

a. Three points are coplanar if they lie on the same plane.

b. Water freezes at temperatures below 32°F.

c. An even number is divisible by 2.

Solution

a. If three points lie on the same plane, then they are coplanar.

b. If water freezes, then the temperature is below 32°F.

c. If a number is even, then it is divisible by 2.

Example 2 ***Writing an Inverse, Converse, and Contrapositive***

Write the (a) inverse, (b) converse, and (c) contrapositive of the following statement.

If the sun is shining, then we are not watching TV.

Solution

a. Inverse: If the sun is not shining, then we are watching TV.

b. Converse: If we are not watching TV, then the sun is shining.

c. Contrapositive: If we are watching TV, then the sun is not shining.

✔ ***Checkpoint*** **Write the (a) inverse, (b) converse, and (c) contrapositive of the conditional statement.**

1. If my allowance increases, then I can save more money.

a. Inverse: If my allowance does not increase, then I cannot save more money.
b. Converse: If I can save more money, then my allowance increased.
c. Contrapositive: If I cannot save more money, then my allowance does not increase.

POINT, LINE, AND PLANE POSTULATES

Postulate 5 Through any two points there exists exactly one line .

Postulate 6 A line contains at least two points .

Postulate 7 If two lines intersect, then their intersection is exactly one point .

Postulate 8 Through any three noncollinear points there exists exactly one plane.

Postulate 9 A plane contains at least three noncollinear points.

Postulate 10 If two points lie in a plane, then the line containing them lies in the plane .

Postulate 11 If two planes intersect, then their intersection is a line .

Example 3 ***Using Postulates and Counterexamples***

Decide whether the statement is *true* or *false*. If it is false, give a counterexample.

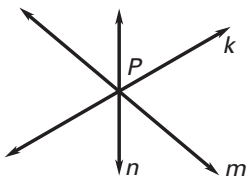
a. A point can lie on more than two lines.

b. Three lines can intersect at no more than three distinct points.

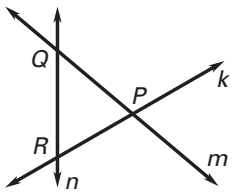
c. If two lines are coplanar, then they intersect.

Solution

a. In the diagram at the right, point *P* is the intersection of line *k*, line *m*, and line *n*. So, it is true that a point can lie on more than two lines.



b. In the diagram at the right, line *k* and line *m* intersect at point *P*, line *m* and line *n* intersect at point *Q*, and line *k* and line *n* intersect at point *R*. There are no more possible intersections. So, it is true that three lines can intersect at no more than three distinct points.



c. In the diagram at the right, line *m* and line *n* are coplanar, but they do not intersect. So, it is false that if two lines are coplanar, then they intersect.