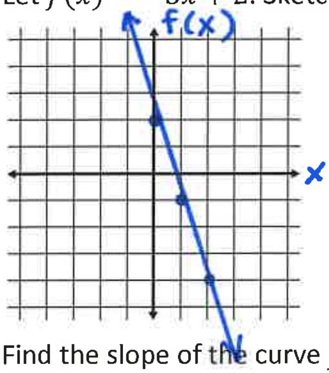
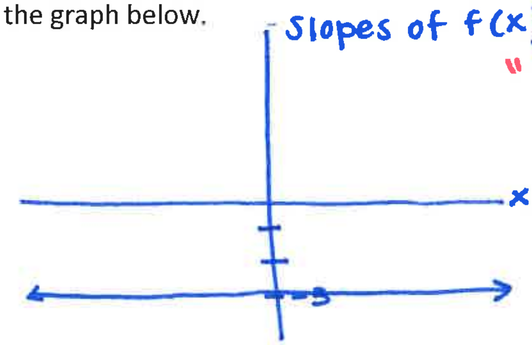
The Making of a Slopes Graph Name: Key

Yesterday, we used limits to calculate the exact slope of a curve at an individual x-value. Today we will calculate many more of these individual slopes and see what happens when we plot them on a graph.

1. Let $f(x) = -3x + 2$. Sketch the graph below.





Slopes of $f(x) = f'(x) = -3$
"the derivative of f"

2. Find the slope of the curve f at $x =$ _____ using the limit definition of slope at a point.

Answers will vary based on what point students were assigned.

3. Use a dot sticker to plot your value from q. 2 on our "slopes" graph. Copy the class graph to the right of your original graph.

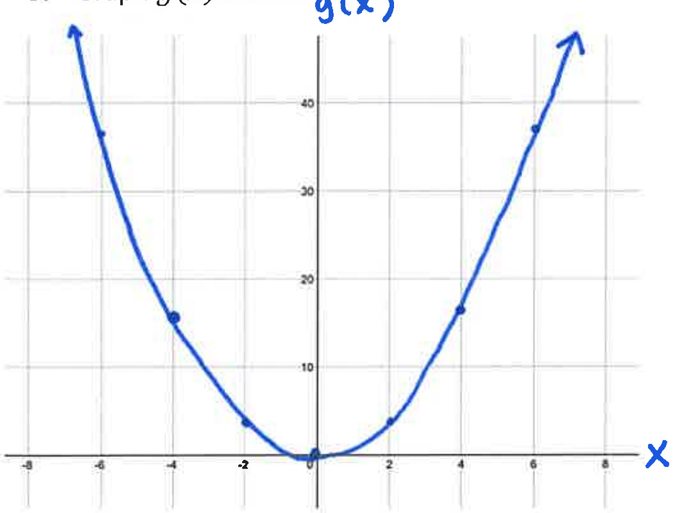
4. What do you think will be the slope of the curve f at $x = 53$? Why do you think so?

$f'(x) = -3$
$f'(53) = -3$

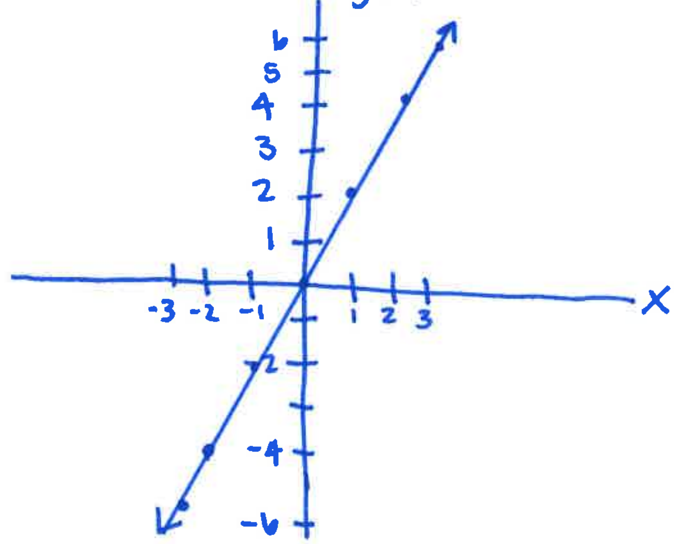
-3; the slope of a line is constant

In questions 5 and 6, we are going to explore the "slopes" graph of two new functions, $g(x) = x^2$ and $h(x) = x^3$.

5. Graph $g(x)$ below.



slopes of g(x) $= g'(x) = 2x$



Let's start by plotting several slope values for $g(x)$.

a. Find the slope of the curve g at x=__________ using the limit definition of slope at a point.

Answers will vary

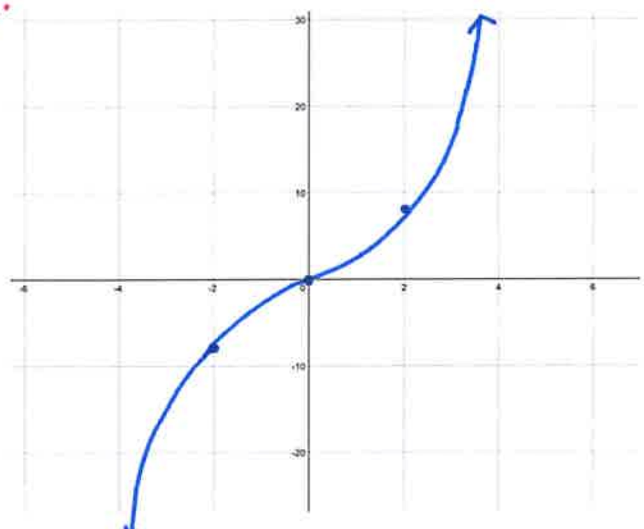
b. Use a dot sticker to plot your value from q. 6a on our "slopes" graph. Copy the class graph to the right of your original graph.

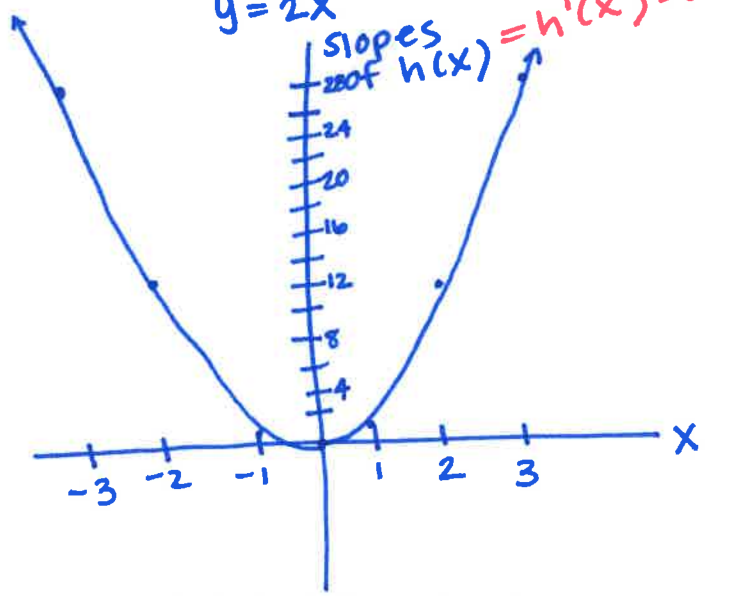
$g'(x) = 2x$
$g'(53) = 106$
The equation for $g'(x)$ helps us find the slope of g at any x-value.

c. What do you think will be the slope of the curve $g(x)$ at $x = 53$? Why do you think so?

$2 \cdot 53 = 106$
The slopes graph looks like $y = 2x$

6. Graph $h(x) = x^3$ below.





Now let's plot several slope values for $h(x)$.

a. Find the slope of the curve $h(x)$ at x=__________ using the limit definition of slope at a point.

Answers will vary

b. Use a dot sticker to plot your value from q. 7a on our "slopes" graph. Copy the class graph to the right of your original graph.

$h(x) = x^3$
$h'(x) = 3x^2$
$h'(53) = 3(53)^2$
$= 8,427$

c. What do you think will be the slope of the curve $h(x)$ at $x = 53$? Why do you think so?

$3(53)^2 = 8,427$
The equation for the slopes is $3x^2$. Plug in 53 for x.

7. What is the purpose of a "slopes" graph?

Helps us see all the slopes at once
The graph can help us find the equation, so we can plug in any x-value + find the slope.

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Topic 2.2—Defining the Derivative

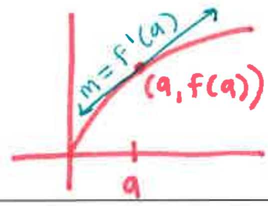
Important Ideas:

① A derivative is a function, $f'(x)$, that gives the slope of the curve at any x-value on $f(x)$.

② Notation for derivative:

- y' – $\frac{dy}{dx}$ – $\frac{d}{dx}y$
- $f'(x)$ – $\frac{df}{dx}$

③ A tangent line touches the curve at one point and shares the slope of the curve.



Equation of tangent line

$y - f(a) = f'(a)(x - a)$

Check Your Understanding!

1. Let $f(x) = 4x^2 - 5$.
   a. Find $f'(x)$ using the definition of the derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{4(x+h)^2-5-(4x^2-5)}{h} = \lim_{h\to 0}\frac{8xh+4h^2}{h} = \lim_{h\to 0}\frac{h(8x+4h)}{h}$$

$$= \lim_{h\to 0} 8x+4h$$

$$\boxed{f'(x) = 8x}$$

   b. What does each point on the graph of $f'(x)$ represent? Be specific.

   Each point represents the slope of the tangent line on $f(x)$ at that x-value.

2. Multiple Choice: $\lim\limits_{h\to 0}\frac{\ln(e+h)-1}{h}$ is

   A. $f'(e)$ where $f(x) = \ln x$
   B. $f'(e)$ where $f(x) = \frac{\ln x}{x}$
   C. $f'(1)$ where $f(x) = \ln x$
   D. $f(1)$ where $f(x) = \ln(x+e)$
   E. $f'(0)$ where $f(x) = \ln x$

   Def. of derivative!!
   $\ln(e+h) = f(x+h)$
   $x = e$, $f(x) = \ln x$
   Asking for deriv. of $\ln x$ @ $x = e$

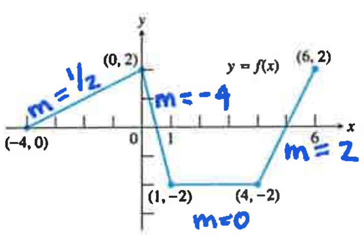
3. The line that is tangent to $q(x)$ at $(-2,7)$ passes through $(5,-1)$. What is $q'(-2)$?

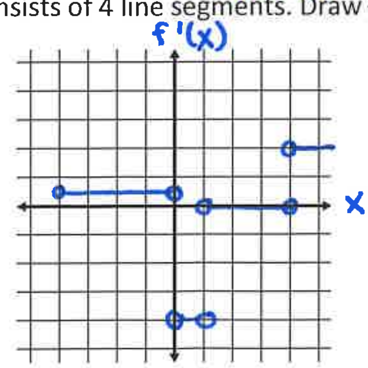
   slope of tangent line = slope of curve $\frac{-1-7}{5-(-2)} = \frac{-8}{7} = q'(-2)$

4. Find the lines that are tangent and normal to the curve $y = x^3$ at $x = 4$.

   $y' = 3x^2$
   at $x = 4$, slope $= 3(4^2) = 48$
   Point $= (4, 4^3) = (4, 64)$
   Tang: $y - 64 = 48(x-4)$
   Normal: $y - 64 = -\frac{1}{48}(x-4)$

5. The graph of $f(x)$, shown below, consists of 4 line segments. Draw $\frac{df}{dx}$ on the coordinate grid.





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