2.4 Reasoning with Properties from Algebra

Goals
- Use properties from algebra.
- Use properties of length and measure to justify segment and angle relationships.

ALGEBRAIC PROPERTIES OF EQUALITY

Let a, b, and c be real numbers.

Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Reflexive Property	For any real number a, $a = a$.
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a can be substituted for b in any equation or expression.

Example 1 *Writing Reasons*

Solve $-2x + 1 = 56 - 3x$ and write a reason for each step.

$-2x + 1 = 56 - 3x$	**Given**
$x + 1 = 56$	Addition property of equality
$x = 55$	Subtraction property of equality

✔ ***Checkpoint*** **Solve the equation. Write a reason for each step.**

1. $12x - 3(x + 7) = 8x$

$12x - 3(x + 7) = 8x$	Given
$12x - 3x - 21 = 8x$	Distr. prop.
$9x - 21 = 8x$	Simplify.
$x - 21 = 0$	Subtr. prop. of =
$x = 21$	Add. prop. of =

Example 2 *Using Properties in Real Life*

Science The Fahrenheit and Celsius temperature scales are related by the formula $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius.

a. Solve the formula for C and write a reason for each step.

b. Use the result to find the Celsius temperatures that correspond to the following Fahrenheit temperatures: 5°F, 32°F, 95°F, 140°F, 212°F. How does the Celsius temperature change as the Fahrenheit temperature changes?

Solution

a.

$F = \frac{9}{5}C + 32$ **Given**

$F - 32 = \frac{9}{5}C$ **Subtraction property of equality**

$\frac{5}{9}(F - 32) = C$ **Multiplication property of equality**

b. Use substitution to find the Celsius temperature that corresponds to 5°F.

$\frac{5}{9}(F - 32) = C$ **Given**

$\frac{5}{9}(5 - 32) = C$ **Substitution property of equality**

$-15 = C$ **Simplify.**

Find the other corresponding temperatures using the same method.

Temperature (°F)	5	32	95	140	212
Temperature (°C)	−15	0	35	60	100

From the table, you can see that the Celsius temperature increases as the Fahrenheit temperature increases.

PROPERTIES OF EQUALITY

	Segment Length	Angle Measure
Reflexive	For any segment AB, $AB = AB$.	For any angle A, $m\angle A = m\angle A$.
Symmetric	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Example 3 *Using Properties of Measure*

Use the information at the right to find $m\angle 1$.

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$
$m\angle 2 + m\angle 3 = m\angle 4$
$m\angle 1 = m\angle 4$

Solution

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360°$	**Given**
$m\angle 2 + m\angle 3 = m\angle 4$	**Given**
$m\angle 1 = m\angle 4$	**Given**
$m\angle 4 + m\angle 4 + m\angle 4 = 360°$	**Substitution property of equality**
$3(m\angle 4) = 360°$	**Simplify.**
$m\angle 4 = 120°$	**Division property of equality**
$m\angle 1 = 120°$	**Transitive property of equality**

✔ ***Checkpoint*** **Complete the following exercise.**

2. In the diagram at the right, B is the midpoint of $\overline{AC}$ and C is the midpoint of $\overline{BD}$. Show that $AB = CD$.

$\overline{AB} \cong \overline{BC}$ Definition of midpoint
$AB = BC$ Definition of congruent segments
$\overline{BC} \cong \overline{CD}$ Definition of midpoint
$BC = CD$ Definition of congruent segments
$AB = CD$ Transitive property of equality