

12.6

The Distance and Midpoint Formulas

- Goals**
- Find the distance between two points.
 - Find the midpoint between two points.

VOCABULARY

Midpoint The midpoint of a line segment is the point on the segment that is equidistant from its endpoints.

THE DISTANCE FORMULA

The distance d between the points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 1 Finding the Distance Between Two Points

Find the distance between $(2, 4)$ and $(4, -3)$.

To find the distance, use the distance formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Write distance formula.} \\ &= \sqrt{(4 - 2)^2 + (-3 - 4)^2} && \text{Substitute.} \\ &= \sqrt{53} && \text{Simplify.} \\ &\approx 7.28 && \text{Use a calculator.} \end{aligned}$$

- ✓ **Checkpoint** Find the distance between two points. Round the result to the nearest hundredth.

1. $(3, 1), (-5, 2)$

8.06

2. $(-4, 5), (6, -2)$

12.21

Example 2 Checking a Right Triangle

Decide whether the points $(1, 0)$, $(3, -1)$, and $(4, 6)$ are vertices of a right triangle.

Solution

Use the distance formula to find the lengths of the three sides.

$$d_1 = \sqrt{(3 - 1)^2 + (-1 - 0)^2} = \sqrt{4 + 1} \\ = \sqrt{5}$$

$$d_2 = \sqrt{(4 - 1)^2 + (6 - 0)^2} = \sqrt{9 + 36} = \sqrt{45}$$

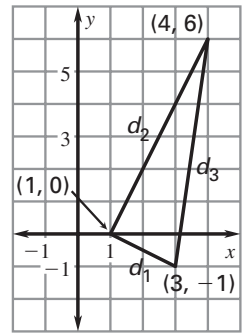
$$d_3 = \sqrt{(4 - 3)^2 + [6 - (-1)]^2} = \sqrt{1 + 49} = \sqrt{50}$$

Next find the sum of the squares of the lengths of the two shorter sides.

$$d_1^2 + d_2^2 = (\sqrt{5})^2 + (\sqrt{45})^2 \quad \text{Substitute for } d_1 \text{ and } d_2. \\ = 5 + 45 \quad \text{Simplify.} \\ = 50 \quad \text{Add.}$$

The sum of the squares of the lengths of the shorter sides is 50, which is equal to the square of the length of the longest side, $(\sqrt{50})^2$.

Answer The given points are vertices of a right triangle.



✔ **Checkpoint** Decide whether the points are vertices of a right triangle.

3. $(1, 0)$, $(7, 0)$, $(7, 5)$

yes

4. $(0, 4)$, $(2, -2)$, $(5, -1)$

yes

THE MIDPOINT FORMULA

The midpoint between (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 3 Finding the Midpoint Between Two Points

Find the midpoint between $(-3, -1)$ and $(2, 4)$. Use a graph to check the result.

Solution

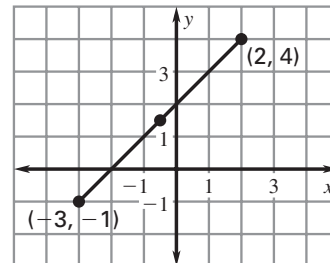
$$\left(\frac{\boxed{-3} + 2}{2}, \frac{\boxed{-1} + 4}{2}\right) = \left(-\frac{\boxed{1}}{2}, \frac{\boxed{3}}{2}\right)$$

Answer The midpoint is $\left(-\frac{1}{2}, \frac{3}{2}\right)$.

Check From the graph, you can see that

the point $\left(-\frac{1}{2}, \frac{3}{2}\right)$ appears halfway

between $(-3, -1)$ and $(2, 4)$. You can also use the distance formula to check that the distances from the midpoint to each given point are equal.



✓ Checkpoint Find the midpoint between the two points.

5. $(3, 5), (-2, -3)$

$$\left(\frac{1}{2}, 1\right)$$

6. $(-7, 4), (5, -10)$

$$(-1, -3)$$

7. $(-9, -2), (6, -2)$

$$\left(-\frac{3}{2}, -2\right)$$

8. $(-6, 9), (2, -5)$

$$(-2, 2)$$