Kuta Software - Infinite Calculus

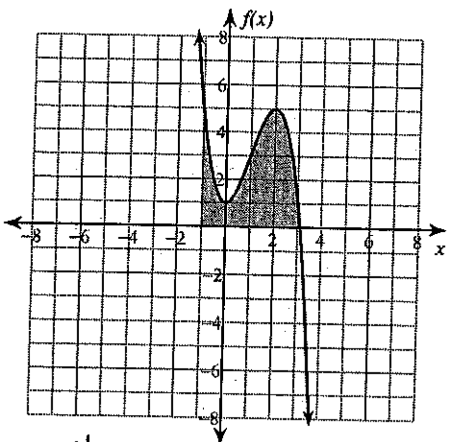
Name________________________

Fundamental Theorem of Calculus

Date________________ Period____

Evaluate each definite integral.

1) $\int_{-1}^{3} (-x^3 + 3x^2 + 1)\, dx$



$= -\frac{x^4}{4} + x^3 + x \Big]_{-1}^{3}$

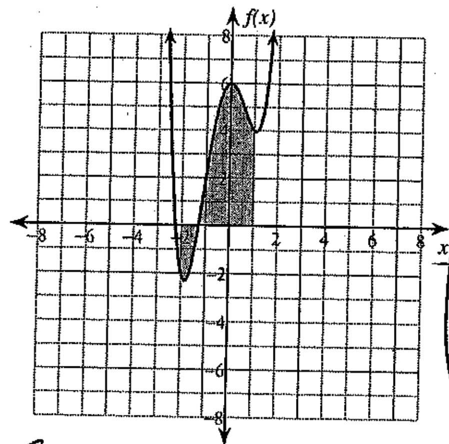
$= \left(-\frac{3^4}{4} + 3^3 + 3\right) - \left(-\frac{1}{4} - 1 - 1\right)$

$= \left(-\frac{81}{4} + \frac{108}{4} + \frac{12}{4}\right) - \left(-\frac{9}{4}\right)$

$= 39/4 + 9/4 = 48/4 = 12$

$\boxed{12u^2}$

2) $\int_{-2}^{1} (x^4 + x^3 - 4x^2 + 6)\, dx$



$\boxed{\frac{177}{20}u^2}$

$= \frac{x^5}{5} + \frac{x^4}{4} - \frac{4}{3}x^3 + 6x \Big]_{-2}^{1}$

$= \left(\frac{1}{5} + \frac{1}{4} - \frac{4}{3} + 6\right) - \left(-\frac{32}{5} + 4 + \frac{32}{3} - 12\right)$

$= \left(\frac{12}{60} + \frac{15}{60} - \frac{80}{60} + \frac{360}{60}\right) - \left(-\frac{96}{15} + \frac{60}{15} + \frac{160}{15} - \frac{180}{15}\right)$

$= \frac{307}{60} - \left(-\frac{56}{15}\right) = \frac{307}{60} + \frac{224}{60} = \frac{771}{60} = \frac{177}{20}$

3) $\int_{1}^{3} (2x^2 - 12x + 13)\, dx$

$\frac{2}{3}x^3 - 6x^2 + 13x \Big]_{1}^{3}$

$\left(\frac{2}{3}(27) - 6(9) + 13(3)\right) - \left(\frac{2}{3} - 6 + 13\right)$

$(18 - 54 + 39) - (7\frac{2}{3})$

$3 - 7\frac{2}{3} = \boxed{-4\frac{2}{3}}$

4) $\int_{0}^{3} (-x^3 + 3x^2 - 2)\, dx$

$-\frac{x^4}{4} + x^3 - 2x \Big]_{0}^{3}$

$= -\frac{81}{4} + 27 - 6 - 0$

$-\frac{81}{4} + \frac{108}{4} - \frac{24}{4} = \boxed{\frac{3}{4}}$

5) $\int_{-1}^{0} (x^5 - 4x^3 + 4x + 4)\, dx$

$\frac{x^6}{6} - x^4 + 2x^2 + 4x \Big]_{-1}^{0}$

$0 - \left(\frac{1}{6} - 1 + 2 - 4\right)$

$-(-2\frac{5}{6}) = \boxed{2\frac{5}{6}}$

6) $\int_{-3}^{0} 4x^{\frac{1}{3}}\, dx$

$4 \cdot \frac{3}{4} x^{4/3} \Big]_{-3}^{0} = 3x^{4/3} \Big]_{-3}^{0}$

$= 0 - \left(3(-3)^{4/3}\right)$

$= -\left(3\sqrt[3]{81}\right)$

$= -\left(3 \cdot \sqrt[3]{3}\sqrt[3]{27}\right.$

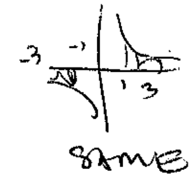
$= -\left(3 \cdot \sqrt[3]{3} \cdot 3\right) = \boxed{-9\sqrt[3]{3}}$

7) $\int_{-4}^{-1} -\frac{4}{x^3}\,dx = \int_{-4}^{1} -4x^{-3}\,dx$

$= -\frac{4}{-2}x^{-2}\Big]_{-4}^{1} = 2x^{-2}\Big]_{-4}^{1} = \frac{2}{x^2}\Big]_{-4}^{1}$

$= \frac{2}{1} - \frac{2}{16} = \frac{30}{16} = \boxed{\frac{15}{8}}$

8) $\int_{-3}^{-1}\frac{4}{x}\,dx = -4\ln x\Big]_{1}^{3}$



$= -4\ln 3 - 0$

$\boxed{= -4\ln 3}$

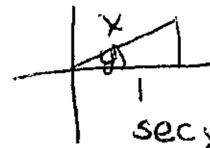
9) $\int_{-\frac{\pi}{4}}^{-\frac{\pi}{6}} 2\cos x\,dx = 2\sin x\Big]_{-\frac{\pi}{4}}^{-\frac{\pi}{6}}$

$= 2\left(\sin -\frac{\pi}{6} - \sin -\frac{\pi}{4}\right)$

$= 2\left(-\frac{1}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right)$

-30 $= 2\left(-\frac{1}{2} + \frac{\sqrt{2}}{2}\right) = \boxed{-1+\sqrt{2}}$

10) $\int_{\sqrt{2}}^{2}\frac{1}{x\sqrt{x^2-1}}\,dx$



$\sec y = \frac{x}{1}$

$\sec y \tan y = \frac{dx}{dy}$

$x\sqrt{x^2-1}$

so $\frac{1}{x\sqrt{x^2-1}} = \sec^{-1}$

$= \sec^{-1} y\Big]_{\sqrt{2}}^{2}$

$= \sec^{-1}(2) - \sec^{-1}(\sqrt{2})$

$= \cos^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\pi}{3} - \frac{\pi}{4} = \frac{4\pi}{12} - \frac{3\pi}{12} = \boxed{\frac{\pi}{12}}$

11) $\int_{-3}^{-2} 5(2x+4)^{\frac{1}{3}}\,dx$

$= 5\cdot\frac{3}{4}\cdot\frac{1}{2}(2x+4)^{\frac{4}{3}}\Big]_{-3}^{-2}$

$= \frac{15}{8}(2x+4)^{\frac{4}{3}}\Big]_{-3}^{-2}$

$= 0 - \frac{15}{8}(-2)^{4/3} = -\frac{15}{8}\sqrt[3]{16} = \boxed{-\frac{15}{4}\sqrt[3]{2}}$

12) $\int_{-1}^{2}\frac{2}{(2x+4)^3}\,dx$ let $u = (2x+4)$, $du = 2\,dx$

$\int_{2}^{8} u^{-3}\,du = \frac{u^{-2}}{-2}\Big]_{2}^{8} = -\frac{1}{2}\left(\frac{1}{8^2} - \frac{1}{2^2}\right)$

$= -\frac{1}{2}\left(\frac{1}{64} - \frac{16}{64}\right)$

$= -\frac{1}{2}\left(-\frac{15}{64}\right) = \boxed{\frac{15}{128}}$

13) $\int_{-1}^{1} e^{2x-2}\,dx = \frac{1}{2}\int_{0}^{4} e^{u}\,du$

let $u = 2x-2$, $du = 2dx$ $\quad \frac{1}{2}e^{u}\Big]_{0}^{4}$

$= \boxed{\frac{1}{2}(1-e^{4})}$

14) $\int_{-4}^{-2}(-x + |-3x-9|)\,dx$

$= \int_{-4}^{-2} -x\,dx + \int_{-4}^{-3}(-3x-9)\,dx + \int_{-3}^{-2}(3x+9)\,dx$

$= -\frac{x^2}{2}\Big]_{-4}^{-2} + \left(-\frac{3}{2}x^2 - 9x\right)\Big]_{-4}^{-3} + \frac{3}{2}x^2 + 9x\Big]$

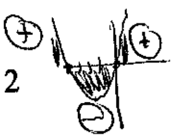
$= -2 - (-8) + \left(-\frac{3}{2}(9) + 27 - \left(-\frac{3}{2}(16) + 36\right)\right)$

$= 6 + 1.5 + 1.5 = \boxed{9}$

15) $\int_{0}^{3} f(x)\,dx,\ f(x) = \begin{cases} \frac{x}{2} - 1, & x \le 2 \\ x^2 - 6x + 8, & x > 2 \end{cases}$

NEG. AREA

POS. ARC.



$\int_{0}^{2}\left(\frac{x}{2} - 1\right)dx + \int_{2}^{3}(x^2 - 6x + 8)\,dx$

$= \frac{x^2}{4} - x\Big]_{0}^{2} + \left(\frac{x^3}{3} - 3x^2 + 8x\right)\Big]_{2}^{3}$

$= \frac{4}{4} - 2 + (9 - 27 + 24) - \left(\frac{8}{3} - 12 + 16\right)$

$+ 6 - \left(\frac{8}{3} + 4\right)$

$= -1 + 6 - 6\frac{2}{3} = -1\frac{2}{3}$ or $\boxed{-\frac{5}{3}}$

16) $\int_{-5}^{1} -|x^2 + 4x|\,dx = -\int_{-5}^{1}|x^2 + 4x|\,dx$

$= \int_{-5}^{-4} + \int_{-4}^{0} + \int_{0}^{1}$

$= -\left(\left[\frac{x^3}{3} + 2x^2\right]_{-5}^{-4} + \left[\frac{x^3}{3} + 2x^2\right]_{-4}^{0} + \left[\frac{x^3}{3} + 2x^2\right]_{0}^{1}\right)$

$-$ (SYM. PROP. $+ \left(\frac{-64}{3} + 32\right) + \frac{1}{3} + 2$)

$-\left(\frac{7}{3} + \left(\frac{-64 + 96}{3}\right) + \frac{7}{3}\right)$

$-\left(\frac{7}{3} + \frac{32}{3} + \frac{7}{3}\right) = -\frac{46}{3}$