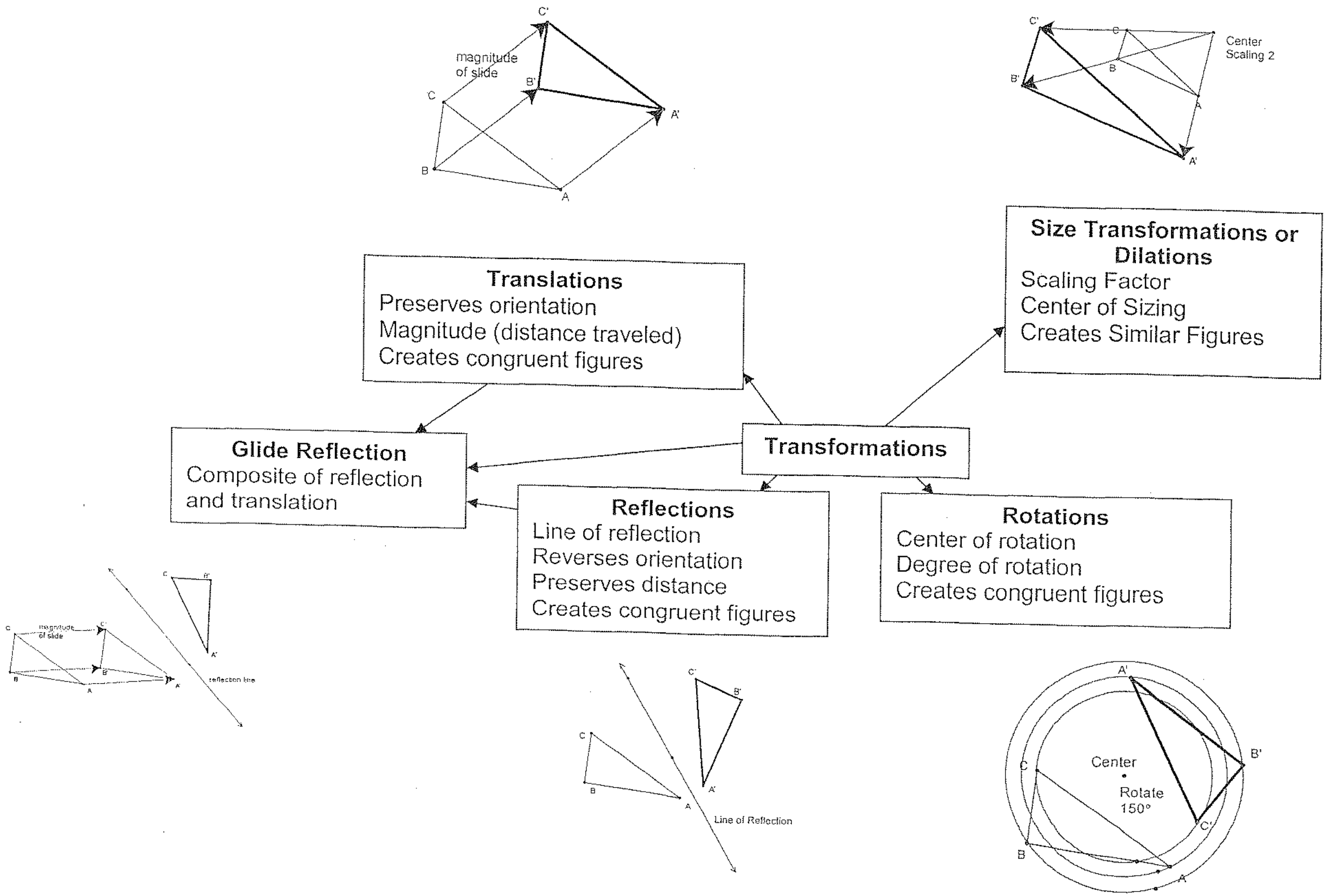
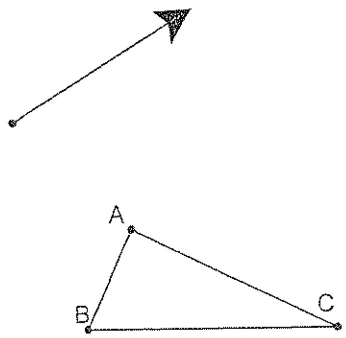
Concept Map for Transformations

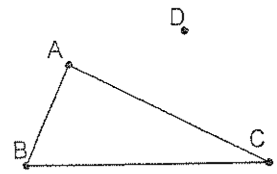


NOTES

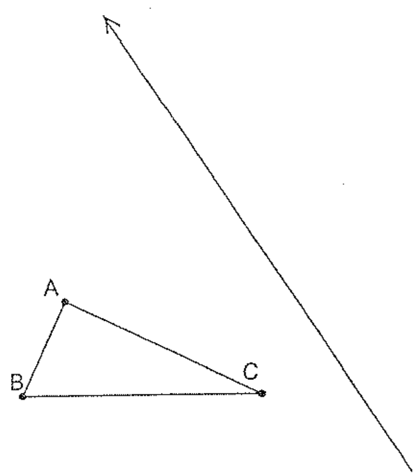
TRANSLATE



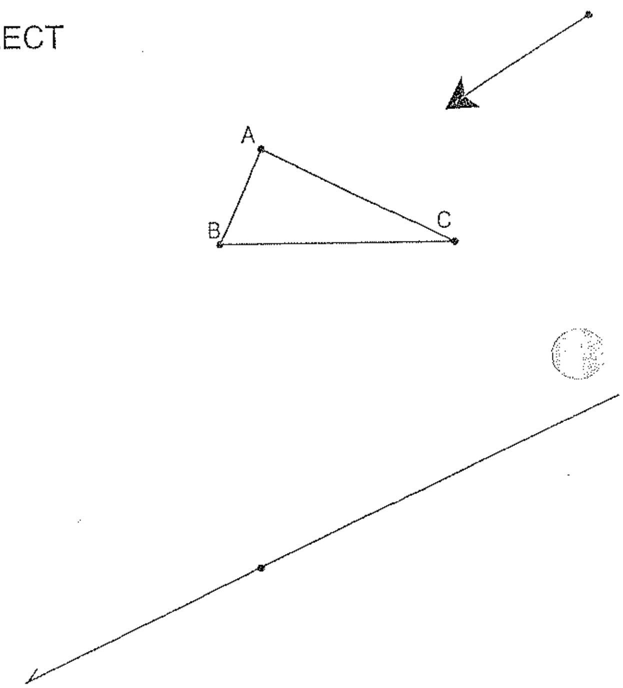
ROTATE -150 DEGREES AROUND D



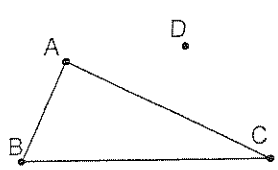
REFLECT



GLIDE REFLECT



SIZE FACTOR 2 CENTER D



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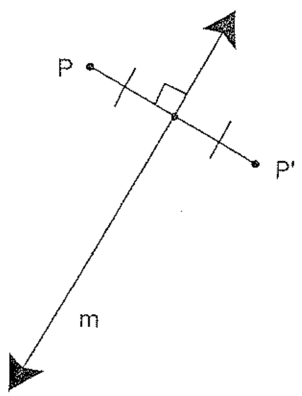
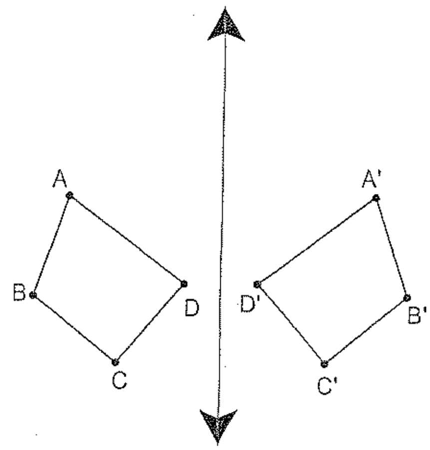
Reflections and Translations

Reflections

If point P is not on line m, the **reflection image** of P over line m is the point P' if and only if m is the perpendicular bisector of $\overline{PP'}$.

If P is on line m, the **reflection image** of P is P itself.

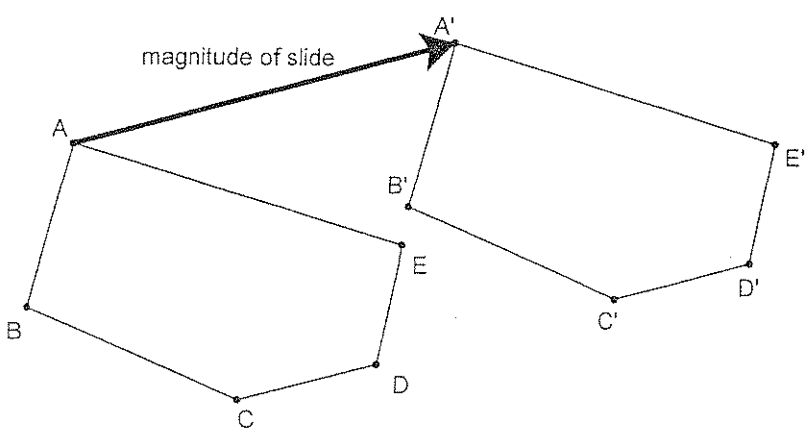




Notice that orientation reverses with convex polygons

Translations

A translation is a transformation which maps a point (x,y) onto a new point (x + a, y + b) where a and b are given real numbers. The distance the point moves is called the **magnitude** of the translation or slide. In this picture, the **length** of the bold arrow is the magnitude.



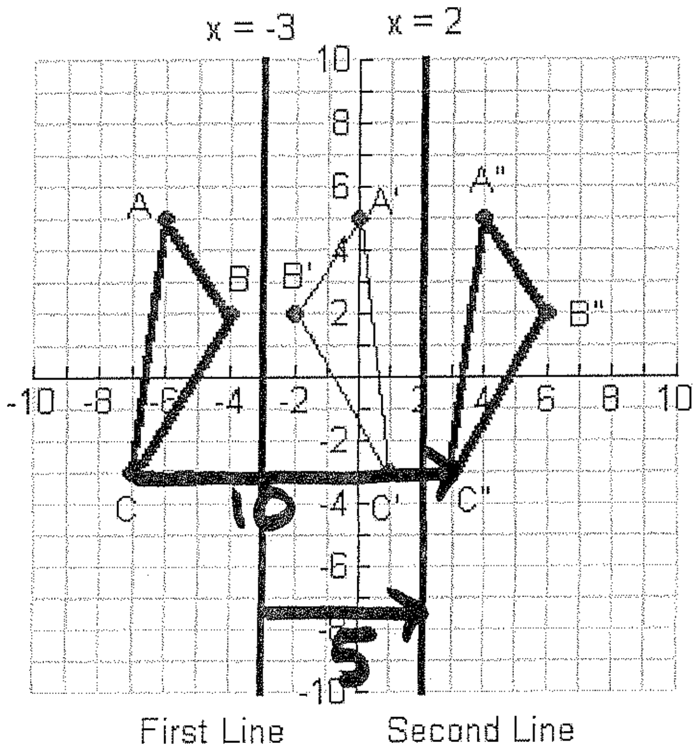
Composite of Two Reflections over Two Parallel Lines

A composite of two reflections over two parallel lines produces a translation.

The **magnitude of the slide** is **double the distance** between the two parallel lines.

The **direction of the slide** is determined by mapping the first line of reflection onto the second line of reflections

Case 1



Composite

$r_{x=2} \circ r_{x=-3}(\Delta ABC)$ OR

$r_{x=2}\,(\,r_{x=-3}(\Delta ABC)\,)$

Image Formula

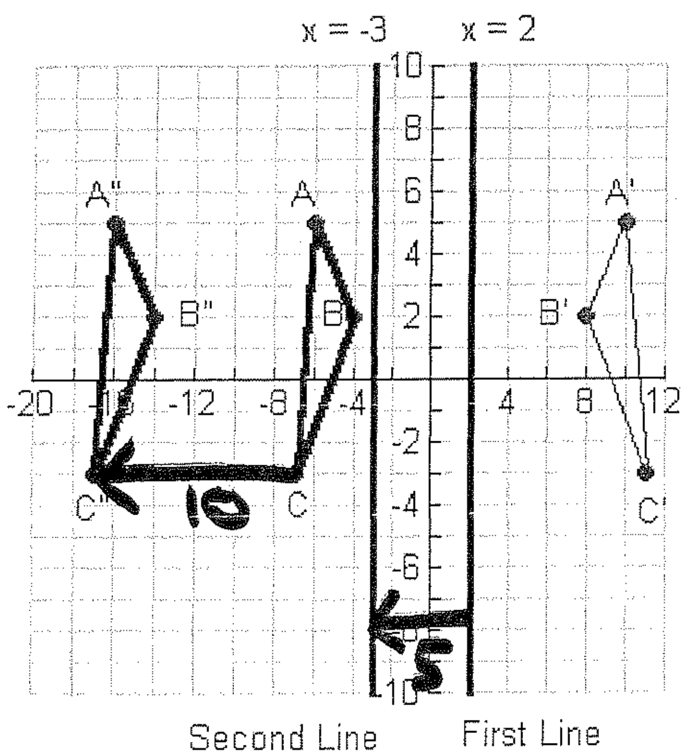
$(x + 10, y)$

Distance between the reflecting lines is 5.

Mapping of x = -3 onto x = 2 moves to the right 5

Therefore, the translation moves to the right 10

Case 2



Composite

$r_{x=-3} \circ r_{x=2}(\Delta ABC)$ OR

$r_{x=-3}\,(\,r_{x=2}(\Delta ABC)\,)$

Image Formula

$(x - 10, y)$

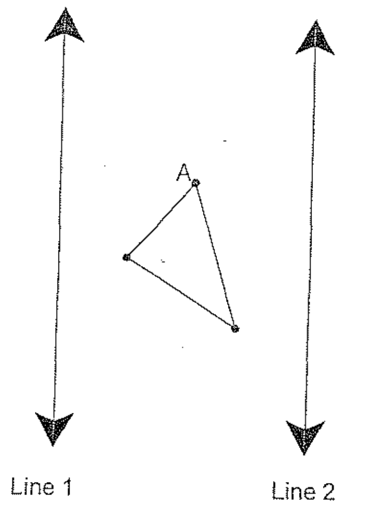
Distance between the reflecting lines is 5.

Mapping of x = 2 onto x = -3 moves to the left 5

Therefore, the translation moves to the left 10 denoted -10

Prove that the composite of two reflections over two parallel lines produces a translation that is double the distance between the lines and in the direction of the mapping of the first line to the second line.

se One – If the point is between the two parallel lines.

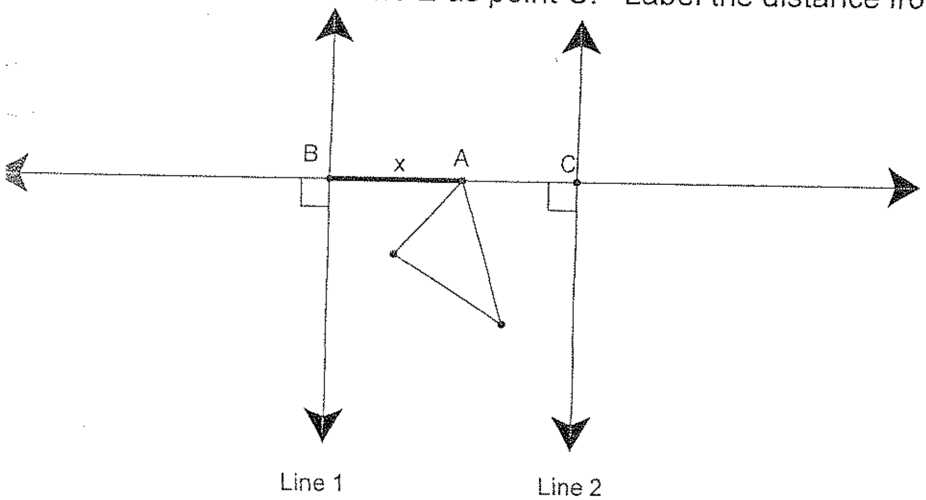


Given: Line 1 ∥ Line 2

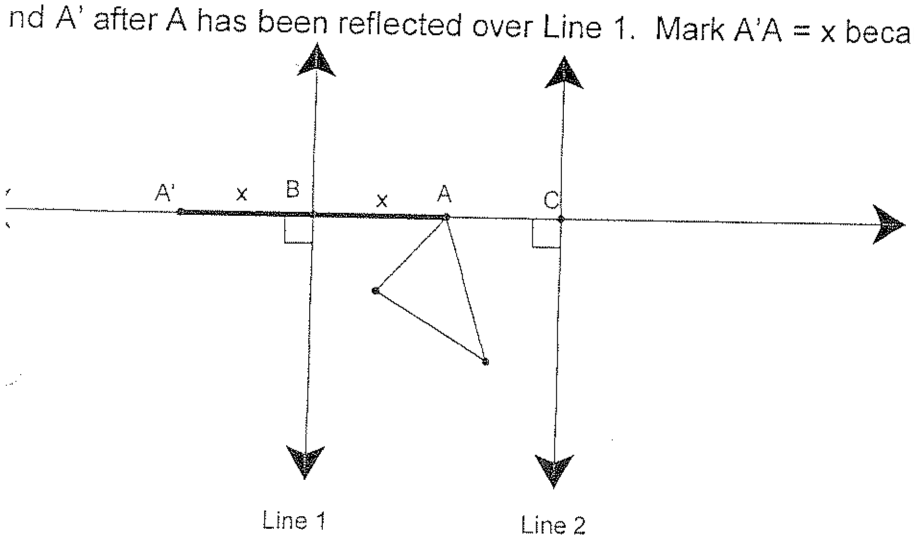
$r_{Line\ 2} \circ r_{Line1}$ (point A)

Proof:

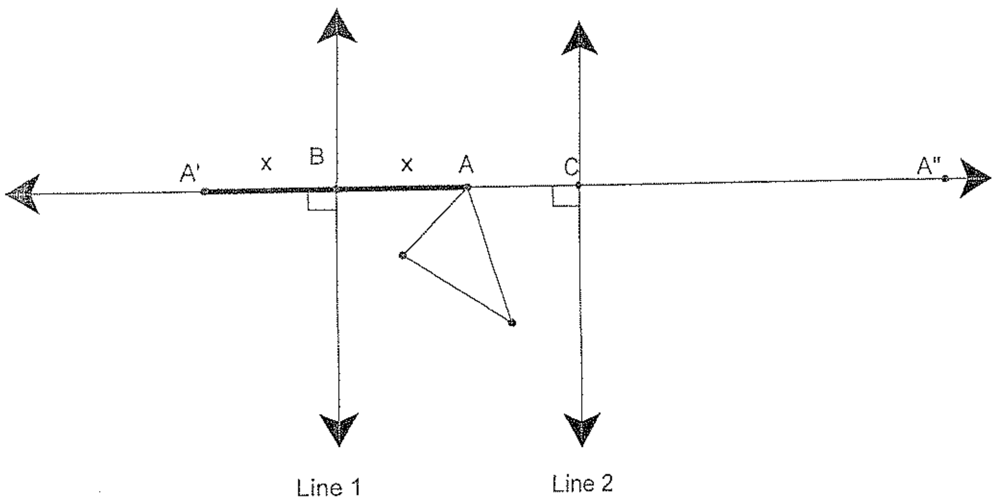
1. Construct a line perpendicular to lines 1 and 2. Since the lines are parallel a line perpendicular to one of the lines would be perpendicular to the other parallel line. Label the intersection with line 1 as point B and the intersection with line 2 as point C. Label the distance from A to B as x; AB = x



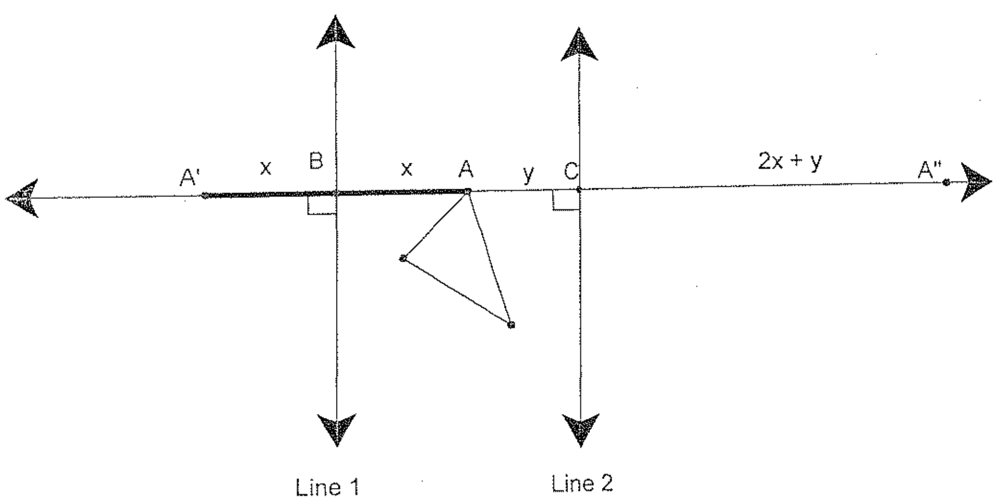
nd A' after A has been reflected over Line 1. Mark A'A = x because reflection maintains distance.



Reflect A' over line 2 and label A"



Since the distance from A to C is unknown, label that distance y; AC = y. Therefore; the distance from C to A" is 2x + y; A"C = 2x + y.



The distance between the two parallel lines is BC = x + y
The distance between A and A" is AA" = 2x + 2y

Therefore, the distance that A translated because of the composite from A to A" is twice the distance between the two lines and the translation moved right because Line 1 moves right to map to Line 2.

You are to prove the other two cases!

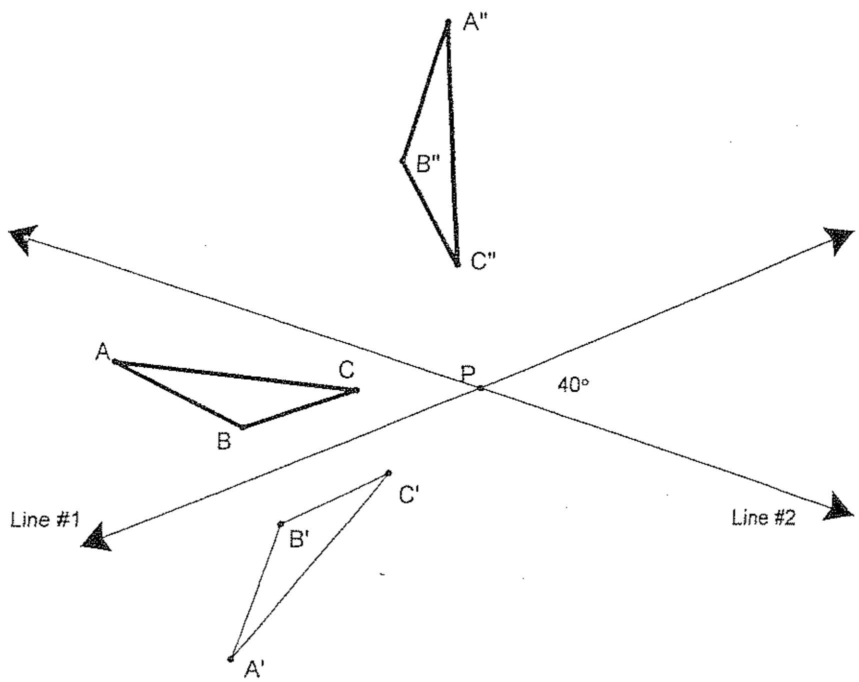
A composite of two reflections over two intersecting lines produces a **rotation**.

The ***center of the rotation*** is the point of intersection of the two intersecting lines

The **magnitude of the rotation** is double the angle formed by the two intersecting lines

The **direction of the rotation** is determined by mapping the first line of reflection onto the second line of reflection

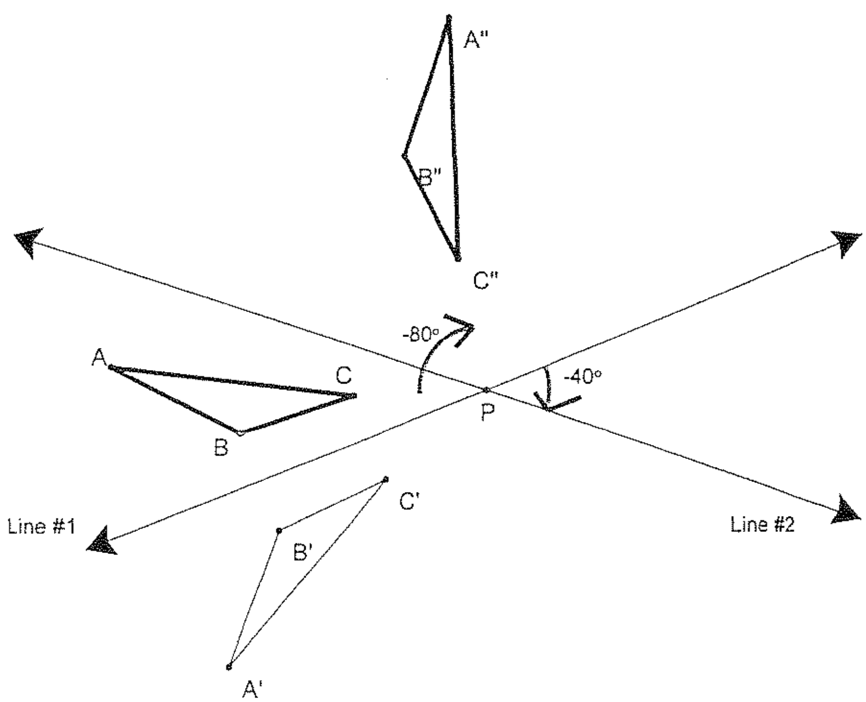
Case #1



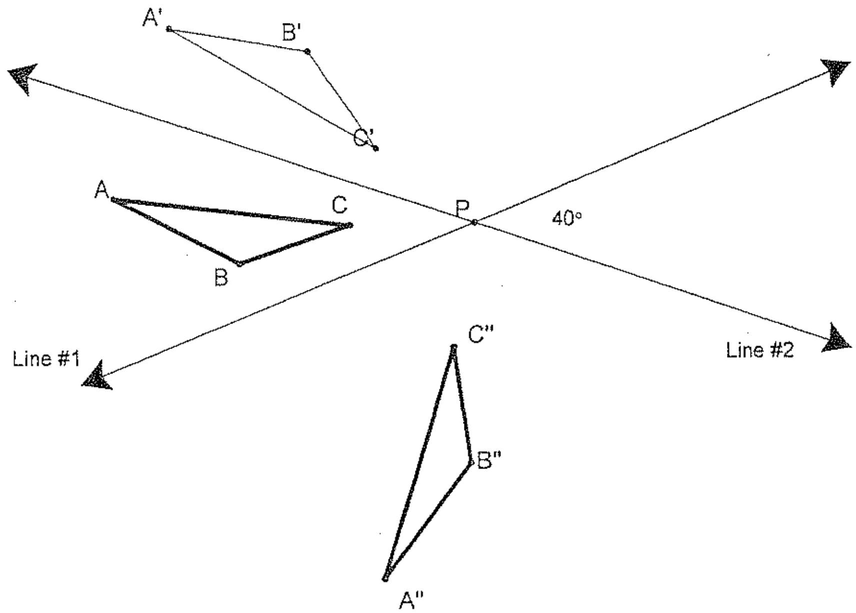
Composite

$r_{Line\ \#2} \circ r_{Line\ \#1}$ (ΔABC)

Since the mapping of Line #1 to Line #2 is -40°, the rotation of ΔABC onto ΔA'B'C' is -80° around center P(the intersection of the two reflecting lines)



Case # 2



Composite

$r_{Line\ \#1} \circ r_{Line\ \#2}$ (ΔABC)

Since the mapping of Line #2 to Line #1 is 40°, the rotation of ΔABC onto ΔA'B'C' is 80° around center P(the intersection of the two reflecting lines)

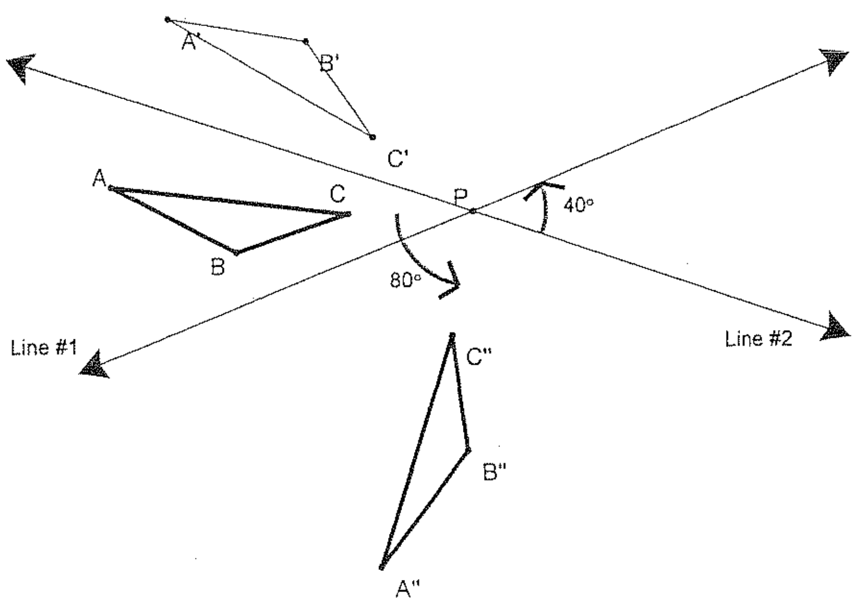
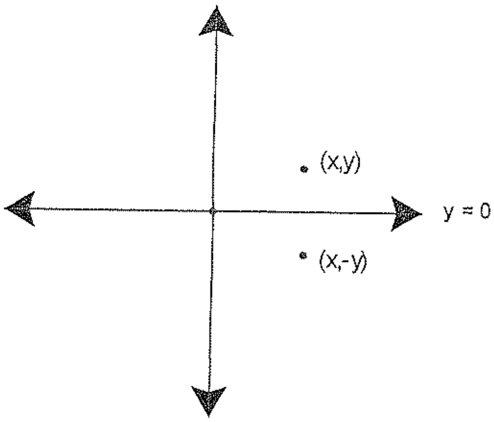
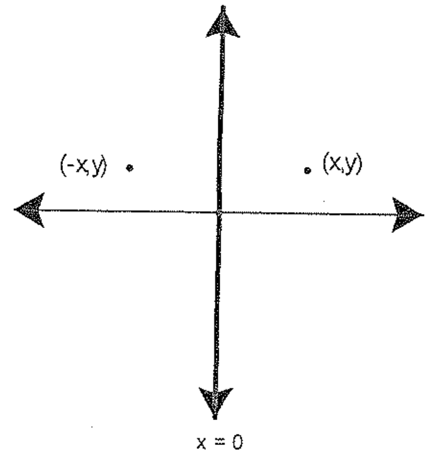


Image Formulas

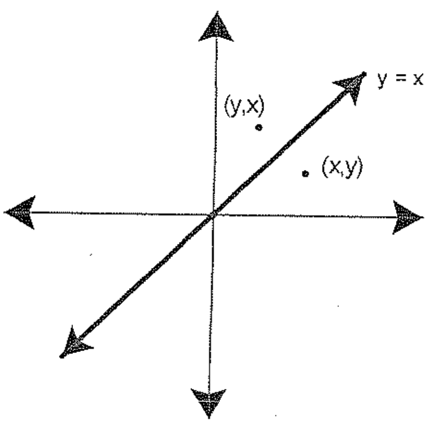
Reflection over the x axis **(x, -y)**



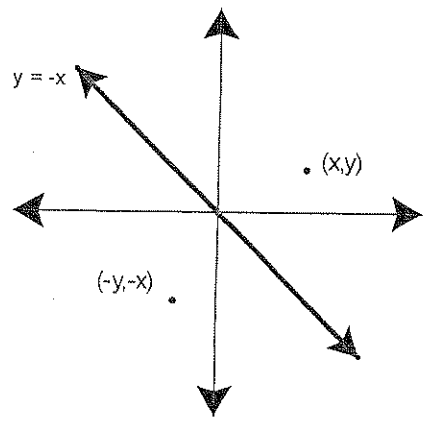
Reflection over the y axis **(-x, y)**



Reflection over the y = x line **(y, x)**

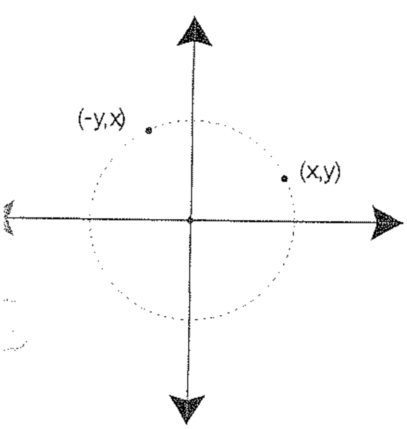


Reflection over the y = -x line **(-y, -x)**



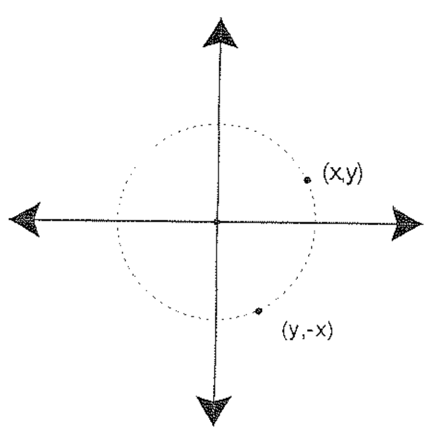
Rotate 90 degrees
Around (0,0)

(-y, x)



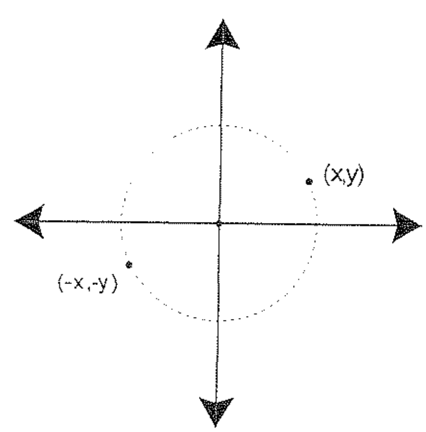
Rotate -90 degrees
around (0,0)

(y, -x)



Rotate 180 degrees or
-180 degrees around (0,0)

(-x, -y)



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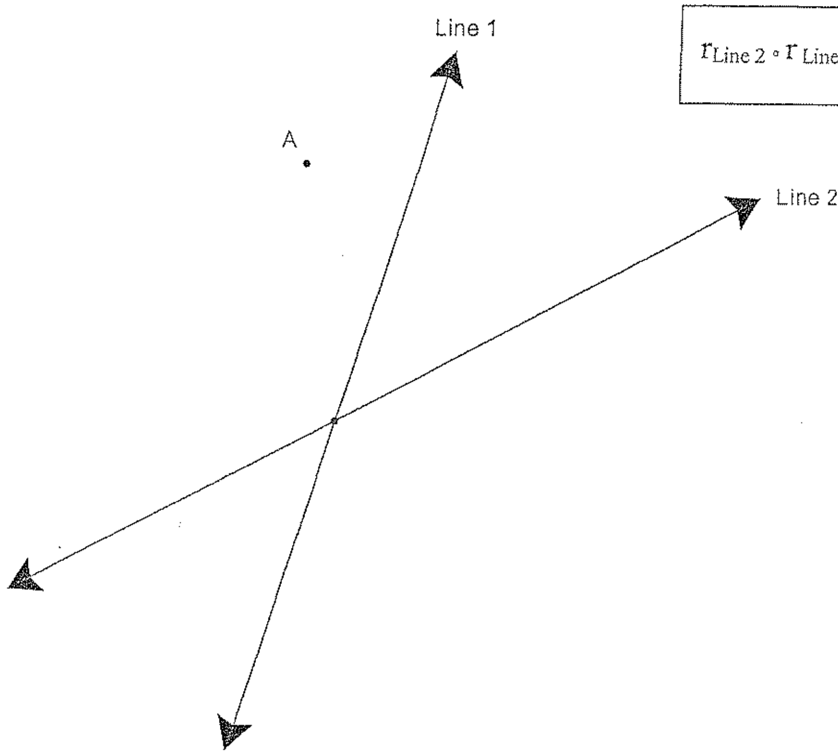
The composite of two reflections over two intersecting lines produces a rotation centered at the intersection of the lines that is double the rotation of the first line to the second line. There are five parts to this proof.

Part One

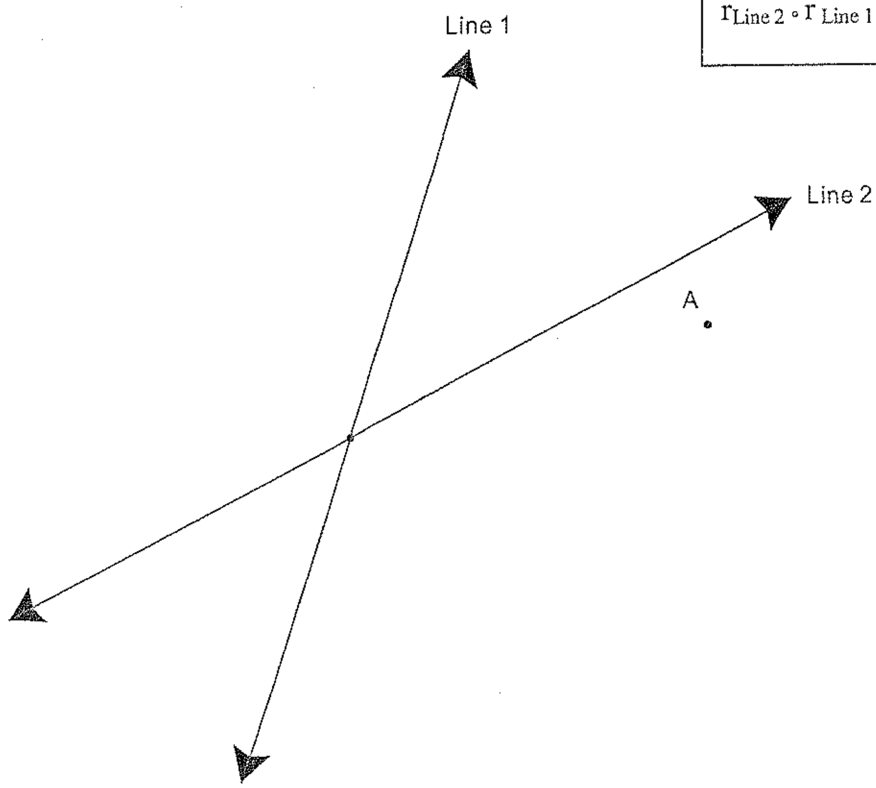
$r_{Line\ 2} \circ r_{Line\ 1}$ (point A)



The composite of two reflections over two intersecting lines produces a rotation centered at the intersection of the lines that is double the rotation of the first line to the second line. There are five parts to this proof.

Part Three

$r_{Line\ 2} \circ r_{Line\ 1}$ (point A)



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