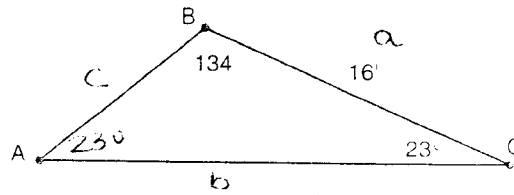
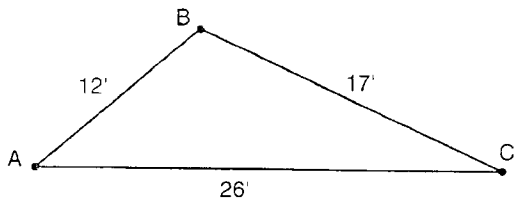


The Law of Sines and Cosines

Name _____

Find the missing part:

ROUND EVERYTHING TO 100THS.



To find m∠A

$$\begin{array}{r} 180 \\ - 134 \\ \hline 46 \\ - 23 \\ \hline 23 \end{array}$$

opp sides Δ so $16' = \overline{AC}$ 23

1. Find measure of angle B

SSS (LAW OF COSINES)

$$b^2 = a^2 + c^2 - 2ac(\cos \angle B)$$

$$26^2 = 17^2 + 12^2 - 2(17)(12) \cos \angle B$$

$$676 = 289 + 144 - 408 \cos \angle B$$

$$243 = -408 \cos \angle B$$

$$\frac{243}{-408} = \cos \angle B$$

$$\cos^{-1}\left(\frac{243}{-408}\right) = m\angle B$$

$$\approx 126.56^\circ$$

2. Find the length of AC

ASA (LAW OF SINES)

$$\frac{\sin(23)}{c} = \frac{\sin(134)}{b} = \frac{\sin(23)}{16}$$

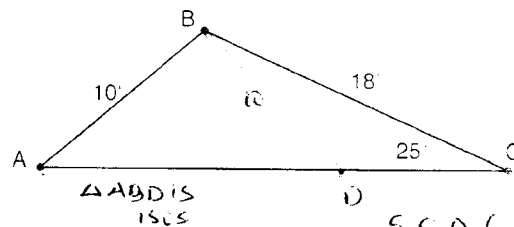
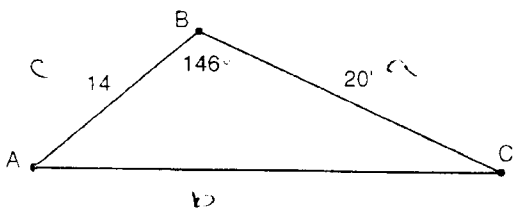
NOT NEEDED

$$16 \cdot \sin(134) = b \sin(23)$$

$$\frac{16 \cdot \sin(134)}{\sin(23)} = b$$

$$\sin(23)$$

$$29.46' \approx m\overline{AC}$$



3. Find the length of AC

SAS (LAW OF COSINES)

$$b^2 = a^2 + c^2 - 2ac(\cos \angle B)$$

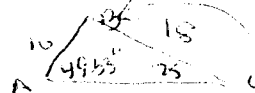
$$= 20^2 + 14^2 - 2(20)(14) \cos(146)$$

$$\approx 32.56'$$

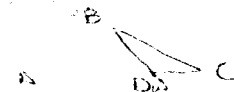
4. Draw the two possible triangles, and find the measure of angle B in each one

SSA (LAW OF SINES)

1ST TRIANGLE IS THE ORIGINAL



2ND TRIANGLE IS ΔBDC



since ΔABD is ISOS

$$m\angle BDA \cong m\angle BDA \cong 49.53$$

$$m\angle BDC + m\angle BDA \text{ (LINEAR PAIR)}$$

$$\text{so } 180 - 49.53$$

$$m\angle CDB \approx 130.47$$

m∠B HERE IS

$$180 - 130.47 - 25$$

$$\approx 24.53^\circ$$

$$\frac{\sin 25}{10} = \frac{\sin A}{18}$$

$$\sin A = \frac{18 \cdot \sin 25}{10}$$

$$A = \sin^{-1}\left(\frac{18 \cdot \sin 25}{10}\right)$$

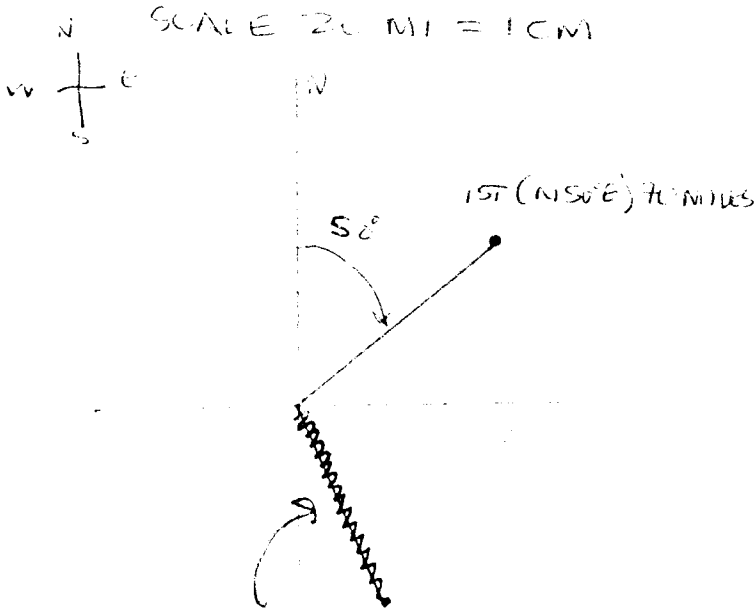
$$\approx 49.53^\circ$$

$$\therefore m\angle B = 180 - 25 - 49.53$$

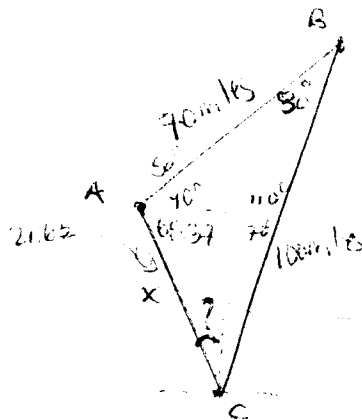
$$\approx 105.47^\circ$$

5. Make a careful drawing using a protractor and ruler. Use a red pencil for the second leg of the trip. SCALE 20 mi = 1 cm

Super Duck decided to go on a trip. Super Duck started out on a heading N 50° E for 70 miles. After traveling the 70 miles, the scenery seemed to be more interesting at a heading of S 20° W so Super Duck changed her direction for the next 100 miles. How far was Super Duck from her starting position? What heading should she take to get back to where she started from? Make a careful drawing using a protractor and ruler. SCALE 20 mi = 1 cm. Estimate your answer and then PROVE MATHEMATICALLY THAT YOU ARE CORRECT!



ESTIMATE THIS
 ≈ 2.6 cm
 OR 52 miles



MATHEMATICALLY PROVE?
 LAW OF COSINE (SAS)
 $m \angle B = 30^\circ$ (ALT. INT. ANG. \cong WHEN LINES ARE \parallel)
 $x^2 = 70^2 + 100^2 - 2(70)(100)\cos 30$
 $x^2 \approx 2775.64431$
 $x \approx 52.68$ miles
 - PRETTY CLOSE!

USE LAW OF COSINES TO FIND $\angle C$

$$70^2 = 2775.64431 + 100^2 - 2(52.68)(100)\cos \angle C$$

$$- 10536.9 = -10536.9 \cos \angle C$$

$$\frac{-10536.9}{-10536.9} = \cos \angle C$$

$$\cos \left(\frac{70^2 + 100^2 - 2(52.68)(100)}{2(52.68)(100)} \right) = \angle C \approx 41.63^\circ$$

PART 2: HEADING S TO GET BACK TO POINT A
 N ? W FOR 52.68 miles
 ≈ 21.63°