3.4 Linear Programming

Goals
- Solve linear programming problems.
- Use linear programming to solve real-life problems.

Your Notes

VOCABULARY

Optimization A process in which you find the maximum or minimum value of some variable quantity

Linear programming The process of optimizing a linear objective function subject to a system of linear inequalities called constraints

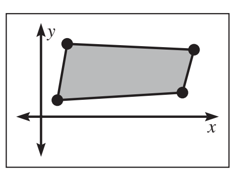
Objective function In linear programming, the linear function that is optimized

Constraints In linear programming, the linear inequalities that form a system

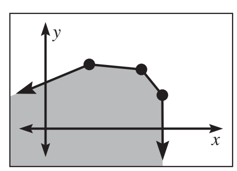
Feasible region In linear programming, the graph of the system of constraints

OPTIMAL SOLUTION OF A LINEAR PROGRAMMING PROBLEM

If an objective function has a maximum or a minimum value, then it must occur at a vertex of the feasible region. Moreover, the objective function will have both a maximum and a minimum value if the feasible region is bounded.



Bounded region



Unbounded region

Your Notes

Example 1 *Solving a Linear Programming Problem*

Find the minimum value and the maximum value of

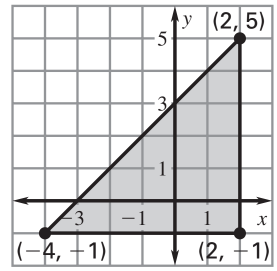
$C = 2y + 3x$ **Objective function**

subject to the following constraints.

$x \leq 2$
$y \geq -1$ **Constraints**
$y - x \leq 3$

Solution

The feasible region determined by the constraints is shown. The three vertices are $(-4, -1)$, $(2, -1)$, and $(2, 5)$. To find the minimum and maximum values of C, evaluate $C = 2y + 3x$ at each of the three vertices.



At $(-4, -1)$: $C =$ $2(-1) + 3(-4) = -14$

At $(2, -1)$: $C =$ $2(-1) + 3(2) = 4$

At $(2, 5)$: $C =$ $2(5) + 3(2) = 16$

The minimum value of C is -14. It occurs when $x =$ -4 and $y =$ -1. The maximum value of C is 16. It occurs when $x =$ 2 and $y =$ 5.

Checkpoint Complete the following exercise.

1. Find the minimum value and maximum value of $C = x - 2y$ subject to the following constraints.

$x \geq 1$
$y \geq x + 1$
$y \leq 7 - x$

minimum: -11, maximum: -3

Your Notes

Example 2 *A Region that is Unbounded*

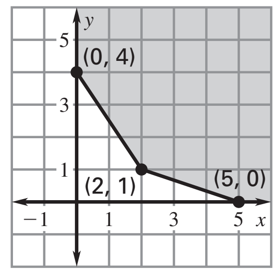
Find the minimum value and the maximum value of

$C = y + 2x$ **Objective function**

subject to the following constraints.

$x \geq 0$
$y \geq 0$ **Constraints**
$2y + 3x \geq 8$
$3y + x \geq 5$

Solution



The feasible region determined by the constraints is shown. The three vertices are (0, 4), (2, 1), and (5, 0). Evaluate $C = y + 2x$ at each of the vertices.

At (0, 4): $C =$ $4 + 2(0) = 4$

At (2, 1): $C =$ $1 + 2(2) = 5$

At (5, 0): $C =$ $0 + 2(5) = 10$

If you evaluate other points in the feasible region, you will see that as the points get farther from the origin, the value of the objective function increases without bound. So, the objective function has no maximum value. The value of the objective function is always at least 4, so the minimum value is 4.

Checkpoint Complete the following exercise.

2. Find the minimum value and the maximum value of $C = 3x + y$ subject to the following constraints.

$x \leq 4$
$x \geq 0$
$y + 2x \geq 9$
$3y - x \geq 6$

minimum: 9, maximum: none

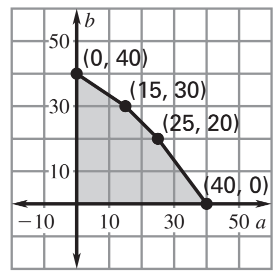
Your Notes

Example 3 *Finding Maximum Profit*

You are making fruit baskets with oranges, bananas, and apples. The table gives the amount of fruit required for the two arrangements. Each day you have 240 oranges, 270 bananas, and 320 apples. Arrangement A earns a profit of \$10 per basket and Arrangement B earns \$8 per basket. How many of each fruit basket should you make per day to maximize your profit?

Fruit	Arrangement A	Arrangement B
Oranges	4	6
Bananas	6	6
Apples	8	6

Solution



Write an objective function. Let a and b represent the number of each type of fruit basket made. Because you want to maximize the profit P, the objective function is: $P = $ ___$10a + 8b$___.

Write the constraints in terms of a and b.

___4___ a + ___6___ $b \leq$ ___240___	Oranges: up to ___240___
___6___ a + ___6___ $b \leq$ ___270___	Bananas: up to ___270___
___8___ a + ___6___ $b \leq$ ___320___	Apples: up to ___320___
$a \geq$ ___0___	Amount cannot be ___negative___.
$b \geq$ ___0___	Amount cannot be ___negative___.

Calculate the profit at each vertex of the feasible region.

At (0, 40): $P = $ ___$10(0) + 8(40) = 320$___

At (15, 30): $P = $ ___$10(15) + 8(30) = 390$___

At (25, 20): $P = $ ___$10(25) + 8(20) = 410$ Maximum___

At (40, 0): $P = $ ___$10(40) + 8(0) = 400$___

Maximum profit is obtained by making ___25___ Arrangement A baskets and ___20___ Arrangement B baskets.

Homework