RC cola !!!!

"ROW YOUR BOAT... FIRST!"

Matrices　　　　　　　　　　　　　　Name______________________

A matrix is a rectangular arrangement of objects, each of which is called an **element** of the matrix. The plural of "matrix" is "matrices."

Matrices are used to store data, describe transformations of various geometric figures and to describe the pictures you see on television screens and computer monitors (these matrices are rectangular arrays of square dots.)

The elements of the matrix are enclosed by large square brackets. The dimension of the matrix is row by column.

A point matrix is always written as a 2 x 1 matrix.

Matrix Multiplication

Matrix multiplication is done by multiplying a row by a column. The matrix on the left is always the row matrix and the matrix on the right is always the column matrix.

In General: Suppose A is an m x n matrix and B is an n x p matrix. Then the product AB is the m x p matrix whose element in row *i* and column *j* is the product of row *i* of A and column *j* of B

Matrix A -daily production number of each style made

worker	#1	#2	#3
▲	8	4	6
●	6	6	8

$$A = \begin{bmatrix} 8 & 4 & 6 \\ 6 & 6 & 8 \end{bmatrix}$$

Matrix B -number of strips of each type of wood for each style

style	oak	walnut	cherry
#1	10	10	0
#2	8	6	6
#3	0	10	10

$$B = \begin{bmatrix} 10 & 10 & 0 \\ 8 & 6 & 6 \\ 0 & 10 & 10 \end{bmatrix}$$

Matrix C has one row for each worker and one column for each type of wood. The numbers in matrix C give information about the number of strips of each type of wood used daily by each worker.

Matrices Continued

Name________________________

Transposing Matrices

Given Matrix A

	Style 1	Style 2	Style 3
Worker Δ		Matrix A	
Worker ⊙			

Matrix A_T Interchanges the rows and columns

	Worker Δ	Worker ⊙
Style 1		
Style 2	Matrix A_T	
Style 3		

You need to transpose matrices sometimes in order to multiply them.

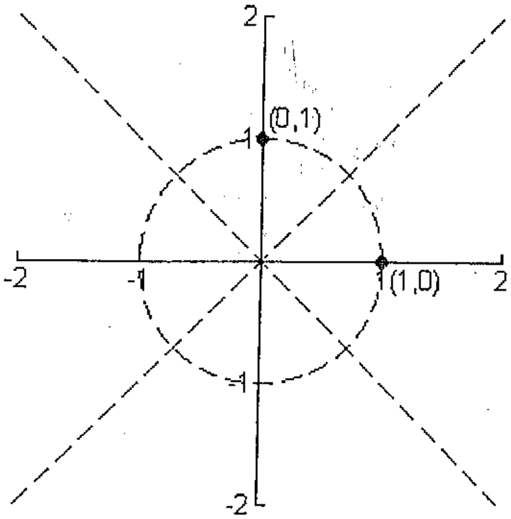
Given:

	Dealer 1	Dealer 2
compact car	18	15
mid-size car	24	17
full-size car	16	20

	Profit
compact car	$800
mid-size car	$1250
full-size car	$2100

2x2 Identity Matrix

Contains the points (1,0) and (0,1)



$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Identity Matrix — Any Point — The Same Point.

Write 2x2 matrices for the following transformations - Give the image formulas

Refection over the x axis

Reflection over the y = -x line

Rotation of 90°

n image formula of (3x + 2y, x – 7y) would have what 2x2 matrix associated with it?

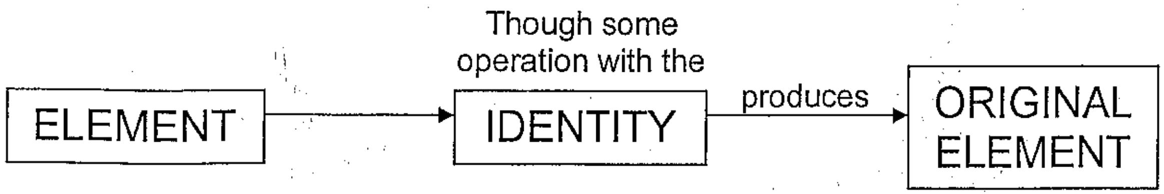
Composites of transformations are done easily with matrices. There order that the composite is written is the order that the matrices are written!

$(r_{y=x} \circ r_{x=0})$ (ΔABC) where A(2,-3) B(4,7) and C(-1,-9)

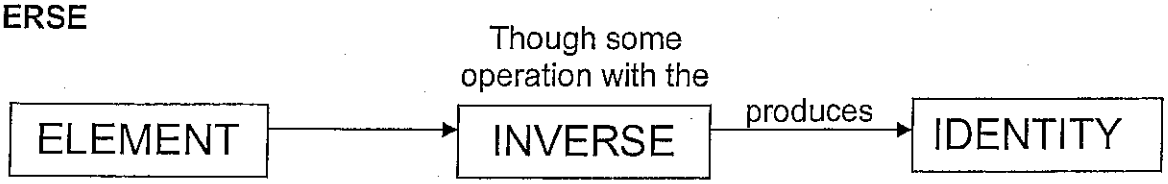
IDENTITIES AND INVERSES

Definitions:

.DENTITY



INVERSE



In the set of real numbers:

Addition: Identity is ________ because 5 + ______ = 5

Inverse is ___________ because 5 + ________ = ________

Multiplication Identity is ________ because 5 ______ = 5

Inverse is ___________ because 5 ________ = ________

In 2x2 matrices for multiplication

Identity is Because

Inverse is Because

Find the identity in the system of *

*	A	B	C	D
A	D	C	A	B
B	C	D	B	A
C	A	B	C	D
D	B	A	D	C

The inverse of A is _______ because

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I.

NAME ______________________

a	f	a	e	g	b	d	c
b	a	b	c	d	e	f	g
c	e	c	d	a	g	b	f
d	g	d	a	e	f	c	b
e	b	e	g	f	c	a	d
f	d	f	b	c	a	g	e
g	c	g	f	b	d	e	a

A. The above table is to be used like a multiplication table. Use the table to give a single letter for each of the following:

1. a * c   2. e * g   3. (d * e) * a   4. d * (e * a)

B. There is an element in the set S = {a, b, c, d, e, f, g} which behaves when used in the "*" operation given by the table similar to the way "0" behaves when real numbers are added, "1" behaves when real numbers are multiplied and $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ behaves when 2X2 matrices are multiplied. What is that element?

. There is an element in set S = {a, b, c, d, e, f, g} that behaves when "starred" with "e" the way ⁻3 behaves when added to 3. What is that element?

. What property of the "*" operation presented in the table guarantees that both * (f * g) and (c * f) * g represent the same element in the set S = {a, b, c, d, e, f, }?

. What is the identity element for the operation "*" presented in the table?

II.

*	a	b	c	d	e	f	g
a	f	a	e	g	b	d	c
b	a	b	c	d	e	f	g
c	e	c	d	a	g	b	f
d	g	d	a	e	f	c	b
e	b	e	g	f	c	a	d
f	d	f	b	c	a	g	e
g	c	g	f	b	d	e	a

A. complete this table of inverses for the {S,*} operational system:

element	* -inverse of that element
a	
b	
c	
d	
e	
f	
g	

B. complete the four numbered equations on the right so that they "mimic" the four numbered equations no the left.

	$4 + x = 11$	$e * x = d$
#1	$\_\_ + (4 + x) = {}^{-}4 + 11$	$\_\_ * (e * x) = a * d$
#2	$({}^{-}4 + \_\_) + x = \_\_$	$(a * \_\_) * x = \_\_$
#3	$\_\_ + x = \_\_$	$\_\_ * x = \_\_$
#4	$x = \_\_$	$x = \_\_$

C. fill in the blanks so that all five equations are equivalent:

$g * x = d$

#1 $\_\_ * (g * x) = d * d$

#2 $(d * \_\_) * x = \_\_$

#3 $\_\_ * x = \_\_$

#4 $x = \_\_$

D. "Solve" the following open sentence by writing a sequence of four equivalent open sentences ending with one of the form " $x =$ 'the solution'."

II. Given this system: $\begin{cases} \ldots \\ 21x+20y=28 \end{cases}$

For equations #1 through #4, give the missing matrix or matrices.

$$\begin{bmatrix}18 & 6\\21 & 20\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}32\\28\end{bmatrix}$$

#1 $\begin{bmatrix} \quad \end{bmatrix}\left(\begin{bmatrix}18 & 6\\21 & 20\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}\right)=\begin{bmatrix}\frac{20}{234} & \frac{^-6}{234}\\\frac{^-21}{234} & \frac{18}{234}\end{bmatrix}\cdot\begin{bmatrix}32\\28\end{bmatrix}$

#2 $\left(\begin{bmatrix}\frac{20}{234} & \frac{^-6}{234}\\\frac{^-21}{234} & \frac{18}{234}\end{bmatrix}\cdot\begin{bmatrix}18 & 6\\21 & 20\end{bmatrix}\right)\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix} \quad \end{bmatrix}$

#3 $\begin{bmatrix} \quad \end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix} \quad \end{bmatrix}$

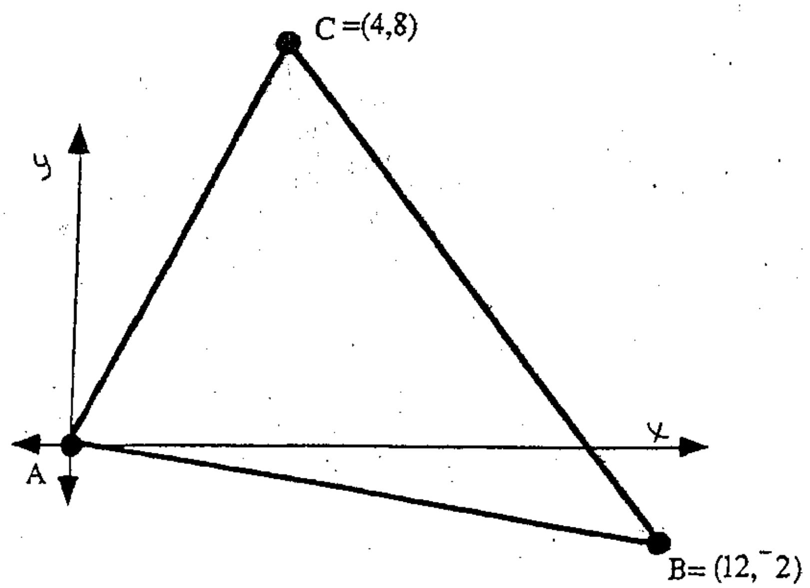
#4 $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix} \quad \end{bmatrix}$

B. What matrix did you produce in part 3 when multiplying $\begin{bmatrix}\frac{20}{234} & \frac{-6}{234}\\\frac{-21}{234} & \frac{18}{234}\end{bmatrix}\begin{bmatrix}18 & 6\\21 & 20\end{bmatrix}$? Does it have a special wonderful name?!

C. What relationship is there between the two matrices listed in part B?

IV Calculate, accurate to the nearest thousandth, the area of triangle ABC in the figure below. This will require several steps. Try to think logically about what to do.

Clearly list each step and explain why you are doing that step!



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The Inverse Matrix and using it to Solve Systems of Equations

The general formula for the inverse matrix of a 2x2 matrix given the original matrix is $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

$$\begin{bmatrix} \frac{D}{AD-BC} & \frac{-B}{AD-BC} \\ \frac{-C}{AD-BC} & \frac{A}{AD-BC} \end{bmatrix}$$

because

$$\begin{bmatrix} \frac{D}{AD-BC} & \frac{-B}{AD-BC} \\ \frac{-C}{AD-BC} & \frac{A}{AD-BC} \end{bmatrix} \bullet \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solve the following system:

96x — 123y = -628.2
47y – 213x = -2622.8

Matrix of Coefficients • Matrix of Variables = Matrix of Constants

$$\begin{bmatrix} 96 & -123 \\ -213 & 47 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -628.2 \\ -2622.8 \end{bmatrix}$$

Inverse Matrix of Coefficients	•	Matrix of Coefficients	•	Matrix of Variables	=	Inverse Matrix of Coefficients	•	Matrix of Constants

Identity Matrix	•	Matrix of Variables	=	Solutions for the values of x and y which satisfy both equations

96x -- 123y = -628.2
47y - 213x = -2622.8

$$\begin{bmatrix} 96 & -123 \\ -213 & 47 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -628.2 \\ -2622.8 \end{bmatrix}$$

The Power of Matrices

Solve this system for the common solution. Show the matrices used and the STEPS!

$5A + 6B - 7C + 8D = -514$
$3C - 12D - 54 = 7A - 8B$
$9B - 13D = 4C - 20$
$16D - 12A = 15B - 17C + 272$

	Coef A	Coef B	Coef C	Coef D
Equation 1	5	6	-7	8
Equation 2	-7	8	3	-1
Equation 3	0	9	-4	-1
Equation 4	-12	-15	17	16

$$\begin{bmatrix} 5 & 6 & -7 & 8 \\ -7 & 8 & 3 & -1 \\ 0 & 9 & -4 & -1 \\ -12 & -15 & 17 & 16 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} -514 \\ 54 \\ -20 \\ 272 \end{bmatrix}$$

$$\begin{bmatrix} A \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} B \end{bmatrix}$$

$$\begin{bmatrix} A \end{bmatrix}^{-1} \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}^{-1} \begin{bmatrix} B \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 14 \\ -32 \\ 24 \\ -28 \end{bmatrix}$$