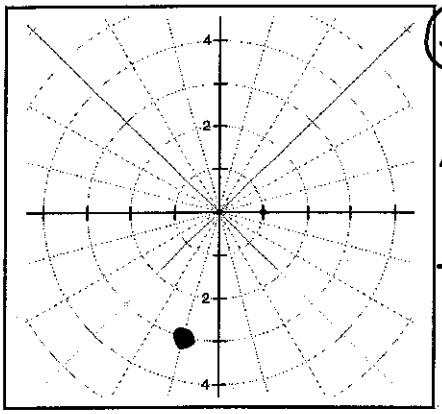
Review Complex Numbers

Name Key

1. Graph the polar point [3, -105°]
   Find its coordinates in a + bi form

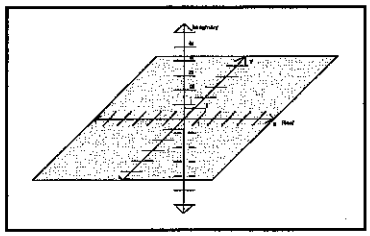


$(3\cos -105, 3\sin -105)$

$\approx (-0.78, -2.90)$

$-0.78 - 2.90i$

2. Graph the complex number (5, -3)
   Find its polar coordinate

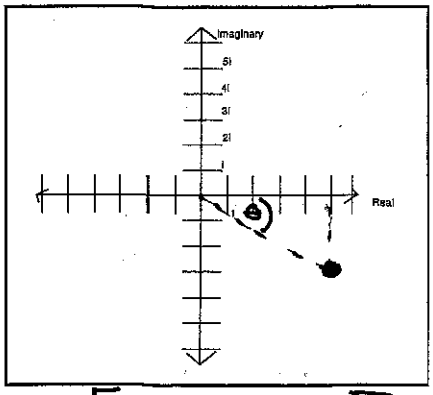


$\sqrt{5^2+3^2}$

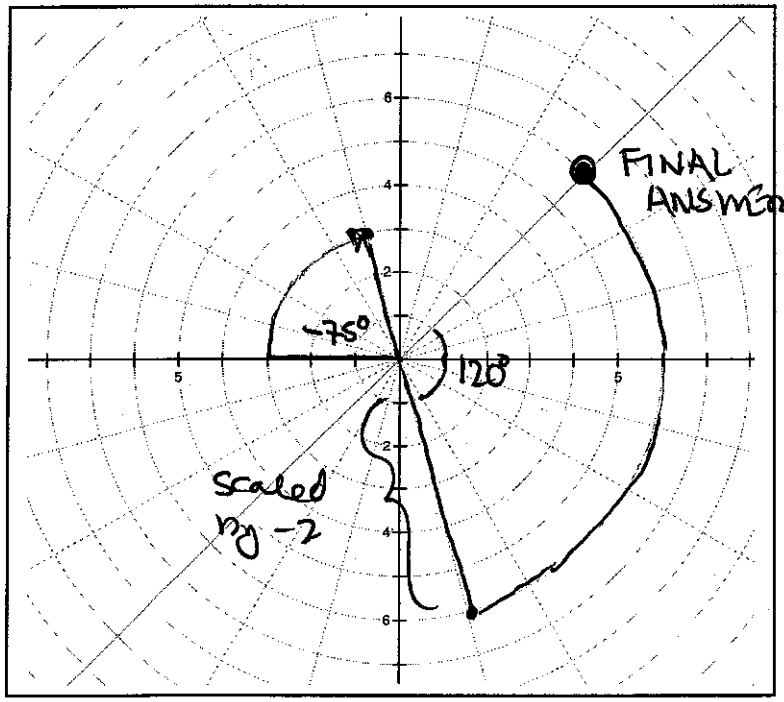
$= \sqrt{25+9}$

$= \sqrt{34}$

$-\tan^{-1}\left(\frac{3}{5}\right) \approx -31°$ $\quad [\sqrt{34}, -31°]$

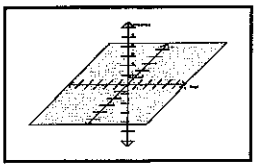


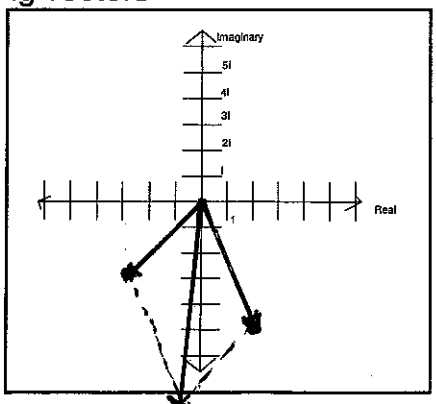
3. Find the product of [-3, -75°] • [-2, 120°] and show that product geometrically.



$[6, 45°]$

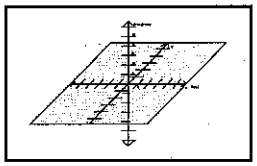
4. Find the sum of (2 + -5i) and (-3 – 3i) and show the sum using vectors

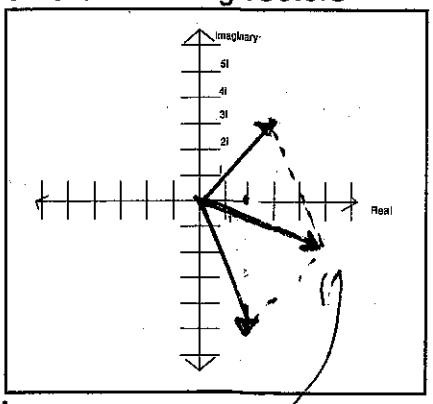




$-1 - 8i$

5. Find the difference of (2 + -5i) and (-3– 3i) and show that difference using vectors





$2 - 5i + (3 + 3i)$

$= 5 - 2i$

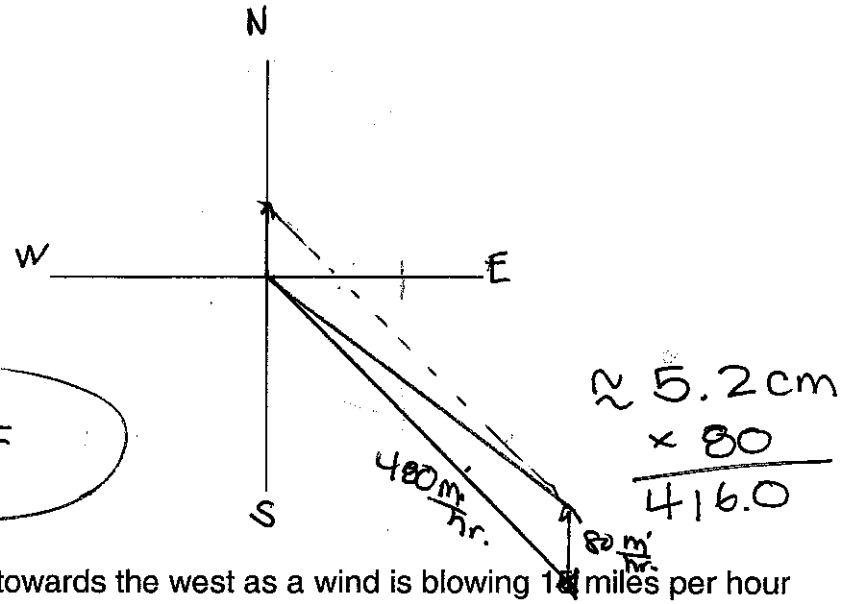
6. An airplane is traveling at a speed of 480 miles per hour going southeast. A wind of 80 miles per hour is going towards the north. What is the resultant speed of the plane? This problem can be thought of as a vector problem. Represent the plane speed with one vector and the wind speed with another. The resultant vector will give you the speed the airplane is actually traveling.

Make a scale
Carefully represent each vector
Complete the parallelogram
Measure the diagonal

$1\text{cm} = 80\frac{\text{mi}}{\text{hr}}$

$\frac{480}{80} = 6$

$\approx 416\frac{\text{mi}}{\text{hr}}$



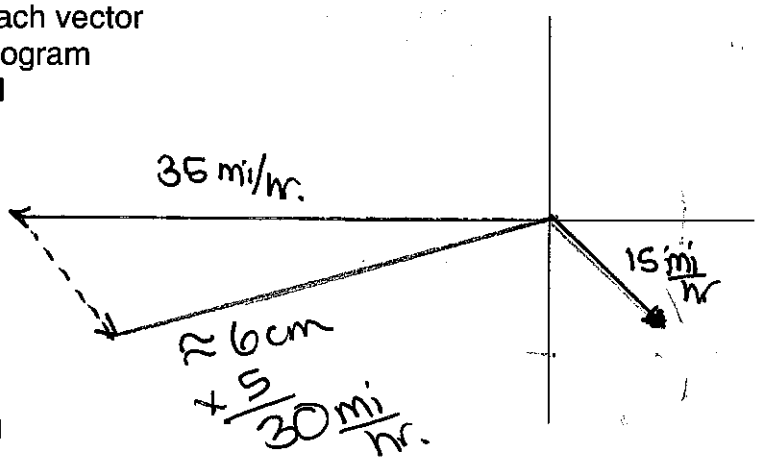
$\approx 5.2\text{cm}$
$\times 80$
416.0

7. A boat is traveling 35 miles per hour towards the west as a wind is blowing 15 miles per hour towards the southeast. How fast is the boat actually going?

Make a scale
Carefully represent each vector
Complete the parallelogram
Measure the diagonal

$1\text{cm} = 5\frac{\text{mi}}{\text{hr}}$

$\approx 30\frac{\text{mi}}{\text{hr}}$



$\approx 6\text{cm}$
$\times 5$
$30\frac{\text{mi}}{\text{hr}}$

8. COMPLEX NUMBERS are written in the form of a + bi where a is the REAL COMPONENT and bi is the IMAGINARY COMPONENT.

9. Different systems:

RECTANGULAR coordinate point is (x,y)

POLAR: coordinate point is [r, θ] where $|r|$ = radius and θ is the degree of rotation based off the x axis

COMPLEX: coordinate point is a + bi or sometimes written (a,b)

Simplify:

10. $(3 + 7i)(-2 + 8i)$

$-6 - 14i + 24i + 56i^2$

$-6 + 10i - 56$

$\boxed{-62 + 10i}$

11. $(3 + 7i) - (-2 + 8i)$

$5 - i$

12. $(3 + 7i) + (-2 + 8i)$

$1 + 15i$

13. i^{456}

$\frac{56}{4} = 14$ R0

so like i^4

or $\boxed{1}$

14. $(3 - 2i\sqrt{3})^2$

$(3 - 2i\sqrt{3})(3 - 2i\sqrt{3})$

$9 - 6i\sqrt{3} - 6i\sqrt{3} + 4i^2 \cdot 3$

$9 - 12i\sqrt{3} - 12$

$\boxed{-3 - 12i\sqrt{3}}$

15. $(3 - 2i)^3$

$(3 - 2i)(3 - 2i)(3 - 2i)$

$(9 - 12i + 4i^2)(3 - 2i)$

$(5 - 12i)(3 - 2i)$

$15 - 10i - 36i + 24i^2$

$\boxed{-9 - 46i}$

16. $8\sqrt{-6}$

$8i\sqrt{6}$

17. $(8\sqrt{-6})(-3\sqrt{-5})$

$8i\sqrt{6} - 3i\sqrt{5}$

$-24i^2\sqrt{30}$

$24\sqrt{30}$

18. $(-3i)^2$

$(-3i)(-3i)$

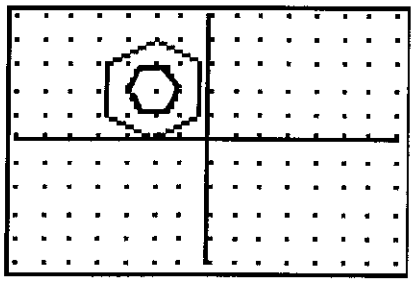
$9i^2$

$\boxed{-9}$

19. $\frac{-2}{i}, \frac{i}{i}$

$= \frac{-2i}{i^2} = \boxed{2i}$

20. How would this graph be made in parametric mode?



$x_1t = \cos t - 2$

$y_1t = \sin t + 2$

$0 \le t \le 36$

$t\text{step} = 60$

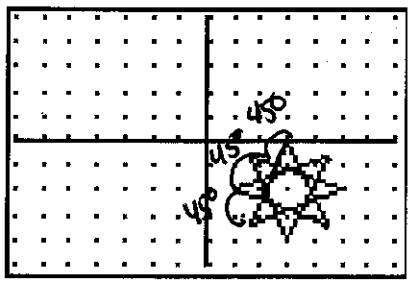
$x_2t = 2\cos t - 2$

$y_2t = 2\sin t + 2$

$90 \le t \le 450$

$t\text{step}: 60$

21. How would this graph be made in parametric mode?



$\frac{360}{8} = 45°$

$x_1t = 2\cos t + 3$

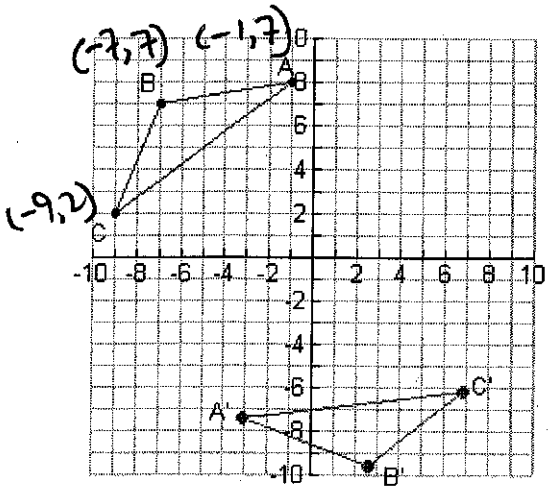
$y_1t = 2\sin t - 2$

$t\text{step}: 45 \cdot 3 = 135$

$0 \le t \le 36 \cdot 3$

1080

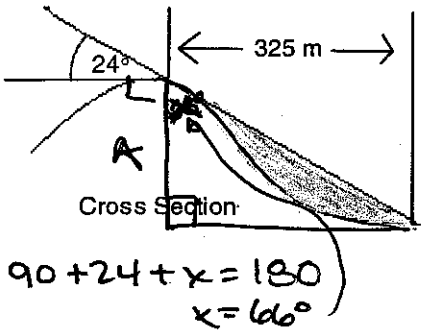
22. Write the two matrices that would be used to transform ΔABC to $\Delta A'B'C'$ and then find the coordinates of $\Delta A'B'C'$ rounded to tenths place.



$$\begin{bmatrix} \cos 150° & -\sin 150° \\ \sin 150° & \cos 150° \end{bmatrix} \cdot \begin{matrix} A & B & C \end{matrix} \begin{bmatrix} -1 & -7 & -9 \\ 8 & 7 & 2 \end{bmatrix}$$

$$\approx \begin{matrix} A' & B' & C' \end{matrix} \begin{bmatrix} -3.1 & 2.6 & 6.8 \\ 7.4 & -9.6 & -6.2 \end{bmatrix}$$

23. Scientists estimate the heights of features on the moon by measuring the lengths of the shadows they cast on the moon's surface. From a photograph, you find that the shadow cast on the inside of a crater by its rim is 325 meters long. At the time the photograph was taken, the sun's angle of elevation from this place on the moon's surface was 24°. How high does the rim rise above the inside of the crater?



$90+24+x=180$

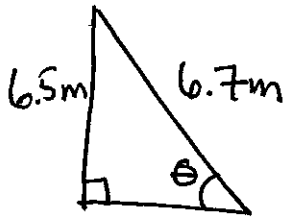
$x=66°$

$\tan(66°)=\frac{325}{x}$

$x=\frac{325}{\tan(66)°}$

$x\approx 144.7\text{ m}$

24. Your cat is trapped on a tree branch 6.5 meters above the ground. Your ladder is only 6.7 meters long. If you place the ladder's tip on the branch, what angle will the ladder make with the ground?

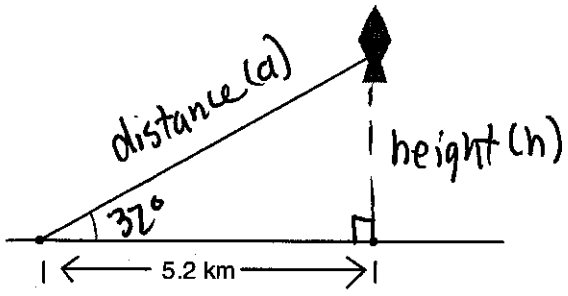


$\sin\theta=\frac{6.5}{6.7}$

$\theta=\sin^{-1}\left(\frac{6.5}{6.7}\right)$

$\approx 75.9°$

25. An observer 5.2 kilometers from the launch pad observes a missile ascending.



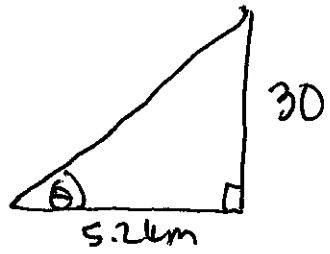
A. At a particular time, the angle of elevation is 32° . How high is the missile? How far is it from the observer?

$\tan 32°=\frac{h}{5.2}$

$h\approx 3.25\text{ km}$

$\cos 32=\frac{5.2}{d}$

$d\approx 6.13\text{ km}$



B. What will the angle of elevation be When the missile reaches 30 kms?

$\tan\theta=\frac{30}{5.2}$

$\theta\approx 80.17°$