MC 5.6-5.9 Connections among f, f', f'' and graphs

1.

x	0	1	2	3	4	5
$f'(x)$	-3	0	-1	5	0	-3
$f''(x)$	5.3	-2.0	1.7	-0.5	1.2	-5.1

Let f be a twice-differentiable function. Selected values of f' and f'' are shown in the table above. Which of the following statements are true?

I. f has neither a relative minimum nor a relative maximum at $x = 1$.
II. f has a relative maximum at $x = 1$.
III. f has a relative maximum at $x = 4$.

(A) I only

(B) II only ✓

(C) III only

(D) I and III only

2. Let f be a function such that $f(-1) = 1$. At each point (x, y) on the graph of f, the slope is given by $\frac{dy}{dx} = -x^2 - xy + y^2 - 1$. Which of the following statements is true?

(A) f has a relative minimum at $x = -1$. ✓

(B) f has a relative maximum at $x = -1$.

(C) f has neither a relative minimum nor a relative maximum at $x = -1$.

(D) There is insufficient information to determine whether f has a relative minimum, a relative maximum, or neither at $x = -1$.

3.

x	0	2	4	6	8	10
$f'(x)$	-1	0	-2	3	0	-1
$f''(x)$	8.333	-1.900	0.971	-0.304	0.400	-4.167

Let f be a twice-differentiable function. Selected values of f' and f'' are shown in the table above. Which of the following statements are true?

I. f has neither a relative minimum nor a relative maximum at $x = 2$.
II. f has a relative maximum $x = 2$.
III. f has a relative maximum $x = 8$.

(A) I only

(B) II only ✓

(C) III only

(D) I and III only

MC 5.6-5.9 Connections among f, f', f'' and graphs

4. Let f be a twice-differentiable function. Which of the following statements are individually sufficient to conclude that $x = 4$ is the location of the absolute maximum of f on the interval $[0, 10]$?

I. $f'(4) = 0$
II. $x = 4$ is the only critical point of f on the interval $[0, 10]$, and $f''(4) < 0$.
III. $x = 4$ is the only critical point of f on the interval $[0, 10]$, and $f(10) < f(0) < f(4)$.

(A) II only
(B) III only
(C) I and II only
(D) II and III only ✓

5. Let f be a function such that $f(1) = 2$. At each point (x, y) on the graph of f, the slope is given by $\frac{dy}{dx} = 5xy - x^2 - y^2 - 5$. Which of the following statements is true?

(A) f has a relative minimum at $x = 1$. ✓
(B) f has a relative maximum at $x = 1$.
(C) f has neither a relative minimum nor a relative maximum at $x = 1$.
(D) There is insufficient information to determine whether f has a relative minimum, a relative maximum, or neither at $x = 1$.

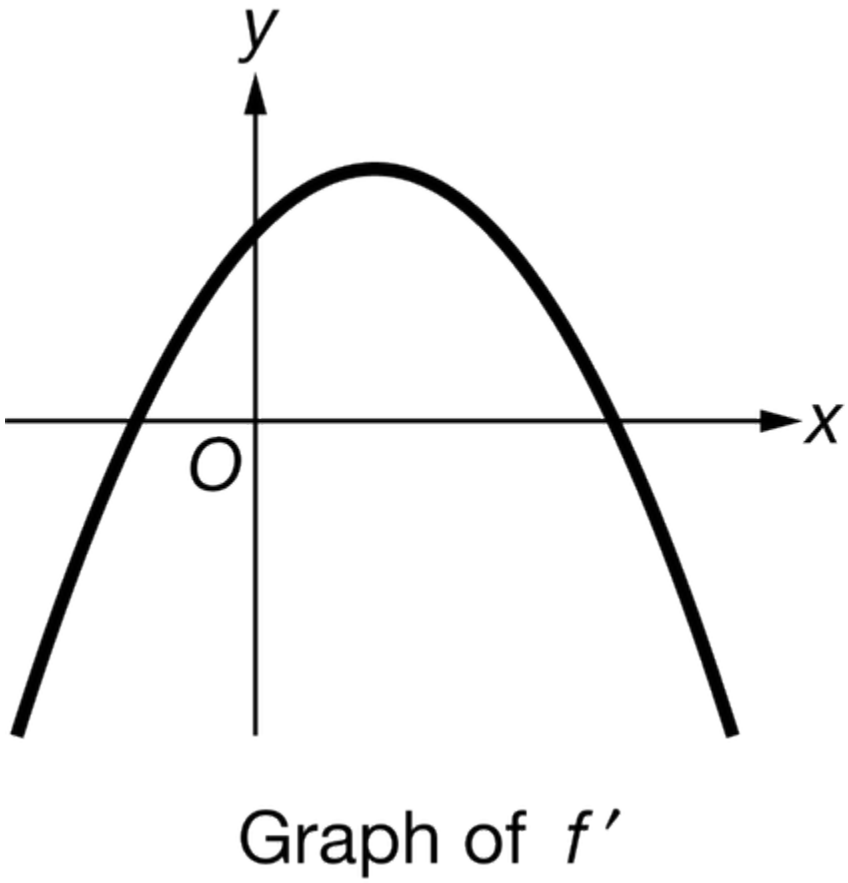
6. Let f be a twice-differentiable function. Which of the following statements are individually sufficient to conclude that $x = 2$ is the location of the absolute maximum of f on the interval $[-5, 5]$?

I. $f'(2) = 0$
II. $x = 2$ is the only critical point of f on the interval $[-5, 5]$, and $f''(2) < 0$.
III. $x = 2$ is the only critical point of f on the interval $[-5, 5]$, and $f(-5) < f(5) < f(2)$.

(A) II only
(B) III only
(C) I and II only
(D) II and III only ✓

MC 5.6-5.9 Connections among f, f', f'' and graphs

7.

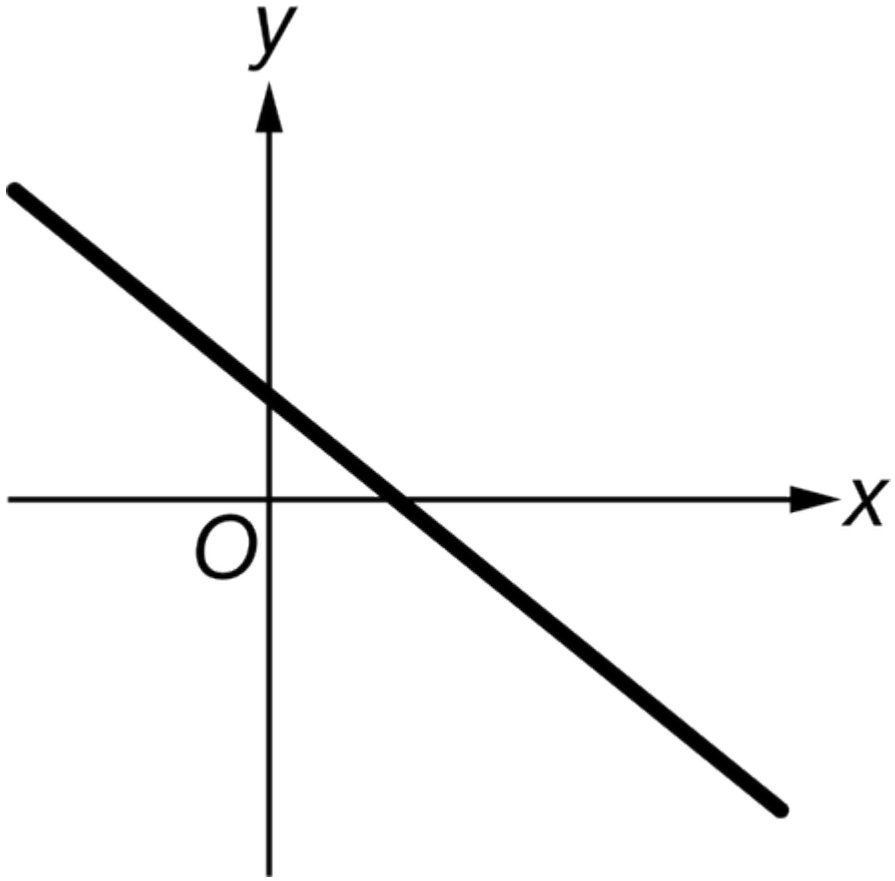


Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f ?

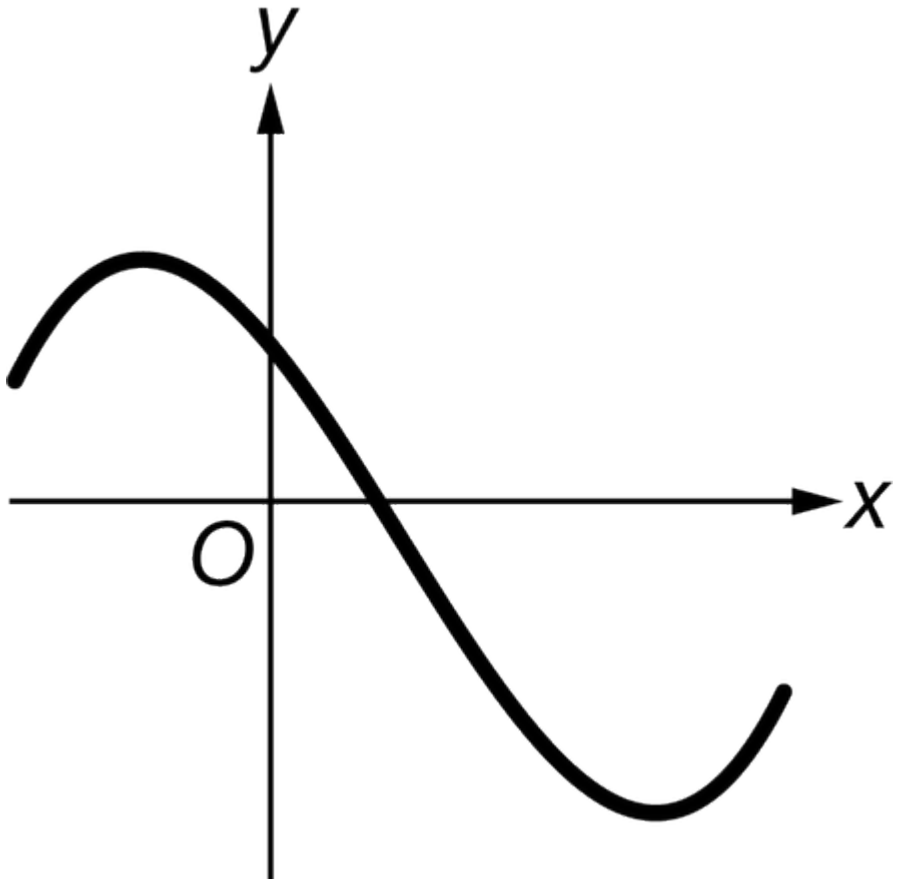
MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)



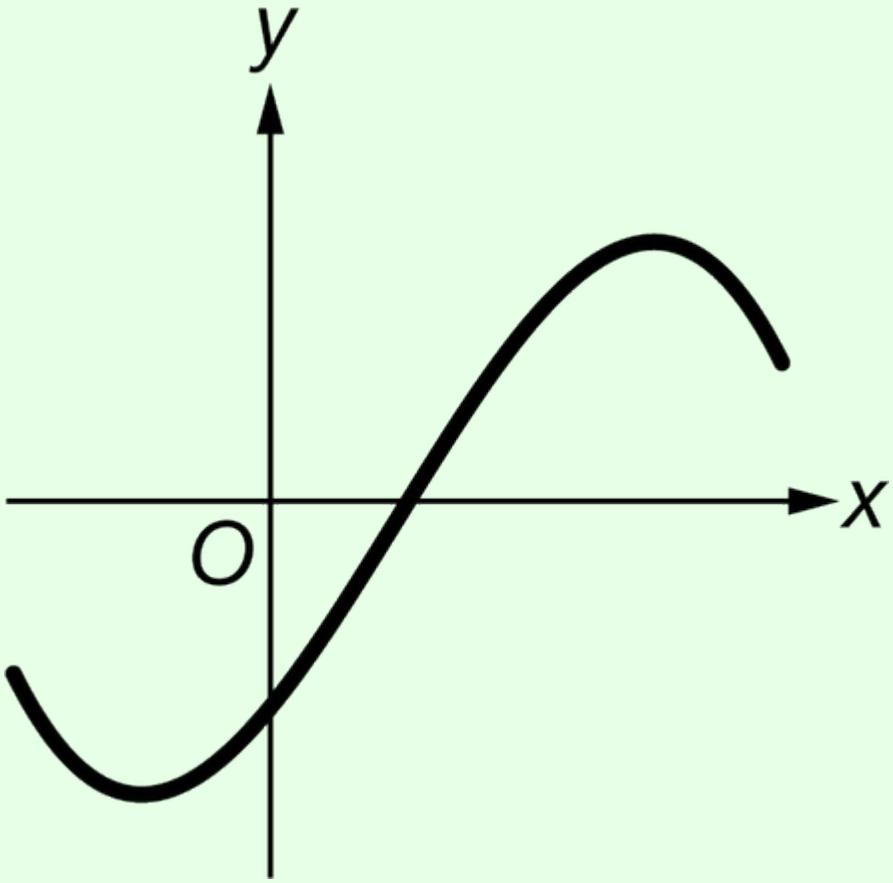
MC 5.6-5.9 Connections among f, f', f'' and graphs

(B)



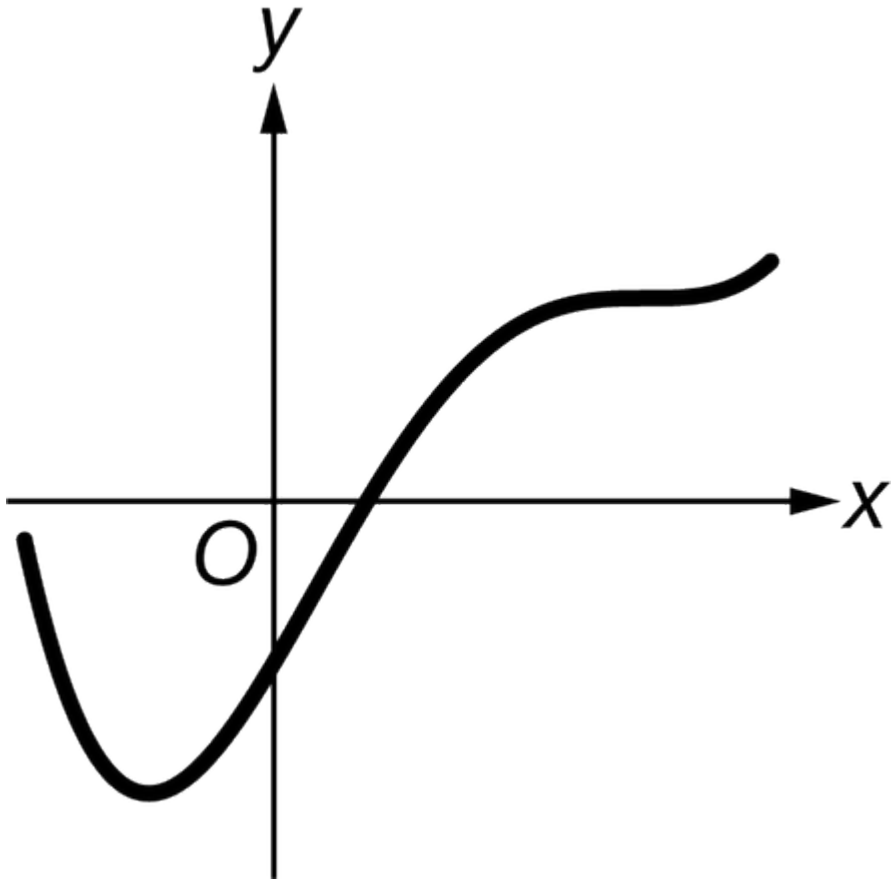
MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)



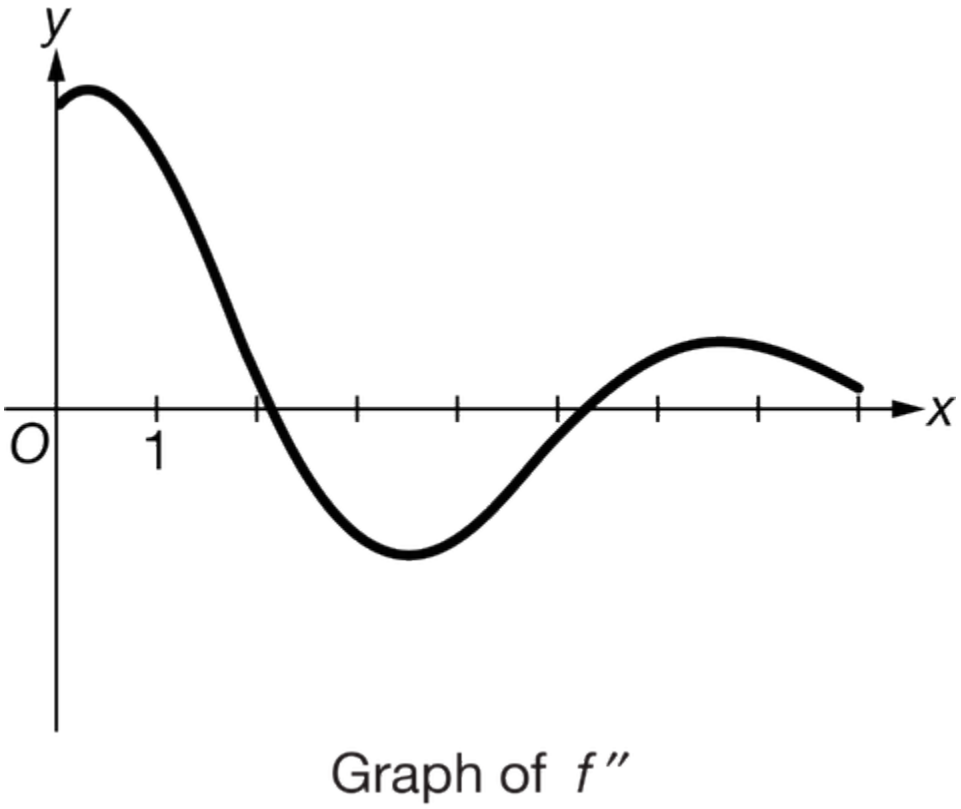
MC 5.6-5.9 Connections among f, f', f'' and graphs

(D)



MC 5.6-5.9 Connections among f, f', f'' and graphs

8.

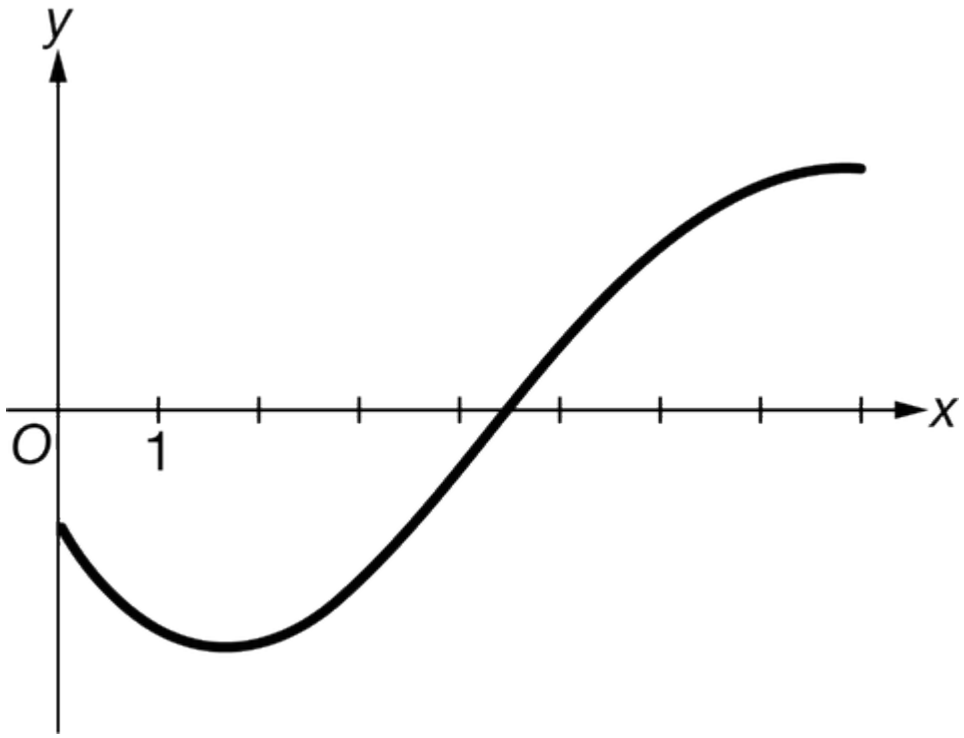


Graph of f''

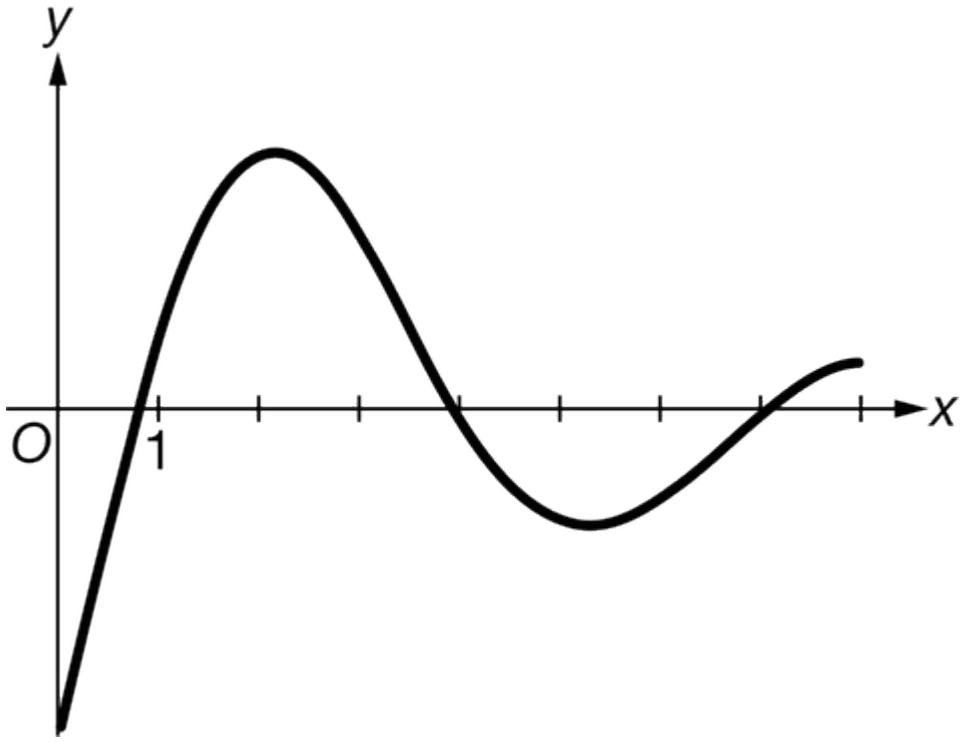
The graph of f'', the second derivative of the function f, is shown above on the interval $0 \leq x \leq 8$. Which of the following could be the graph of f ?

MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)

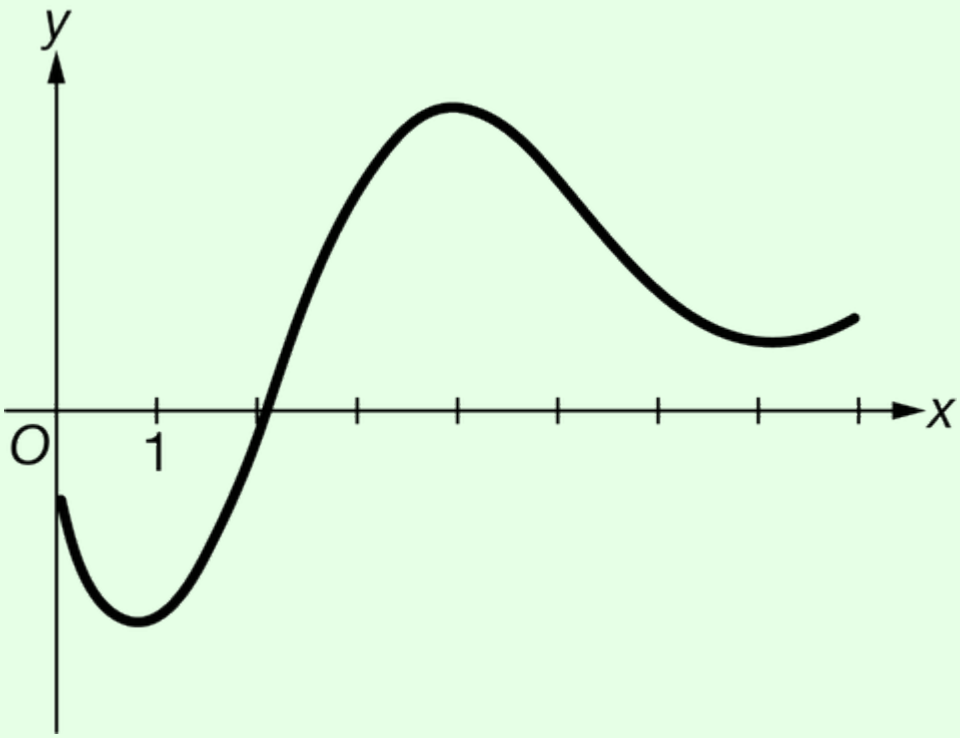


(B)

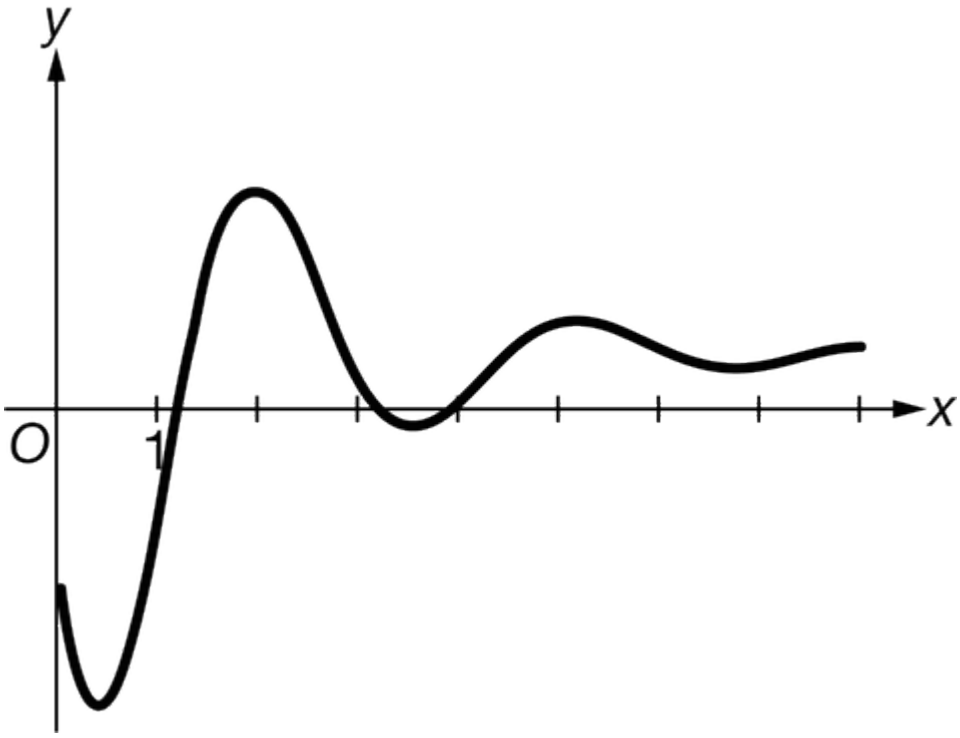


MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)

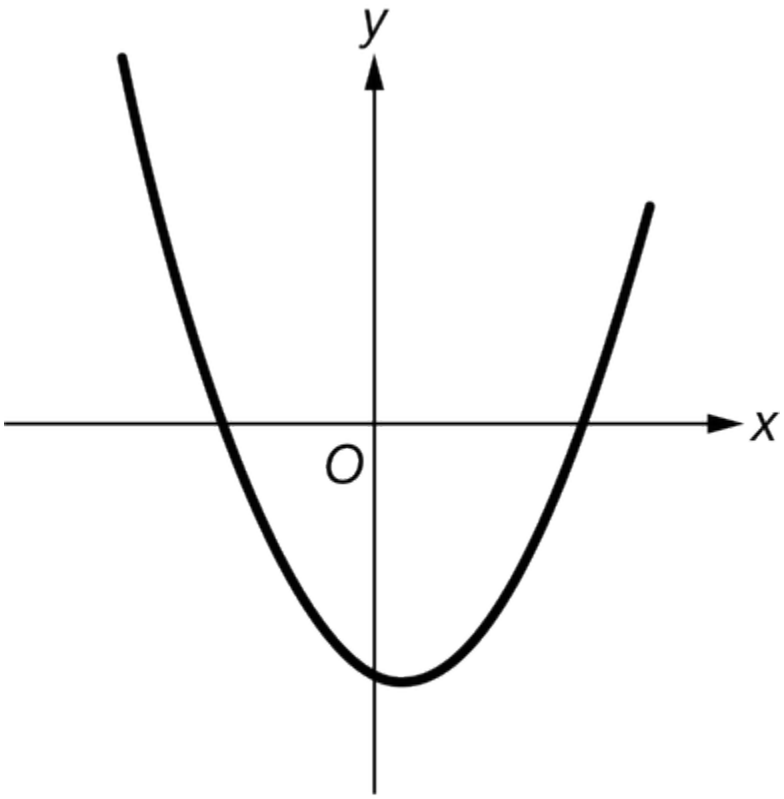


(D)



MC 5.6-5.9 Connections among f, f', f'' and graphs

9.

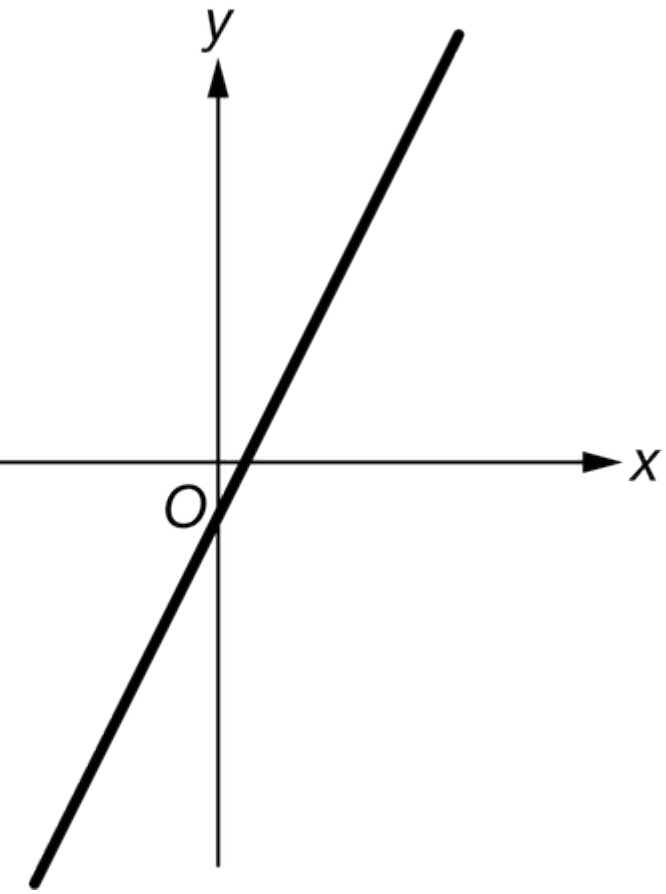


Graph of f'

The graph of f', the derivative of the function f, is shown above. Which of the following could be the graph of f ?

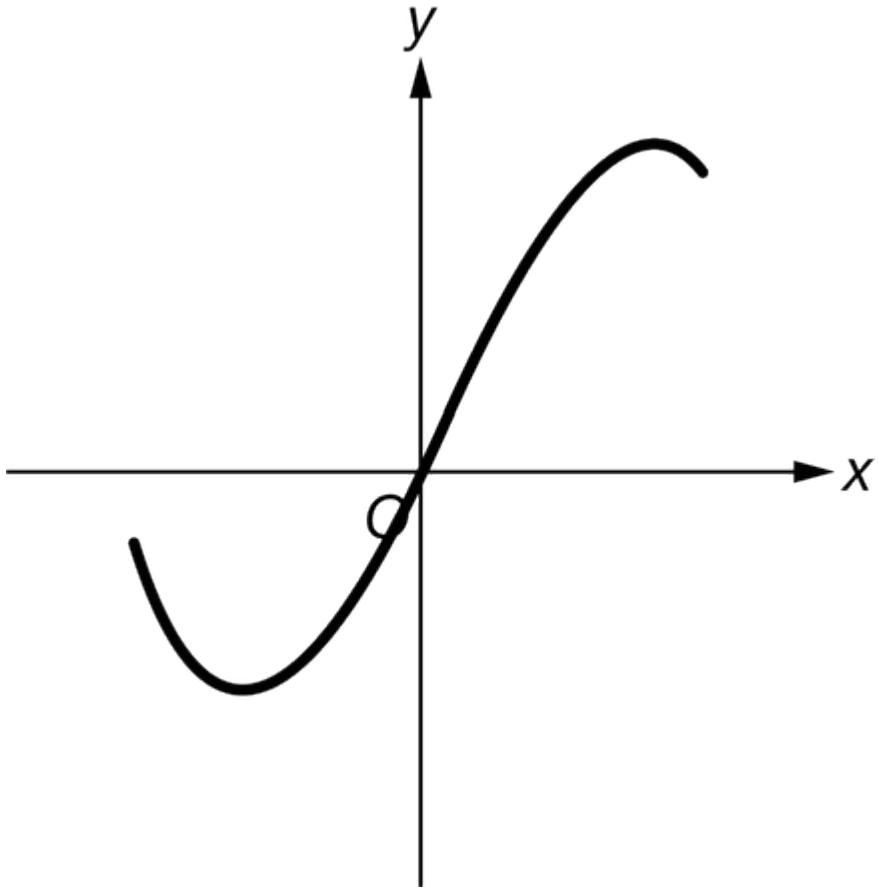
MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)



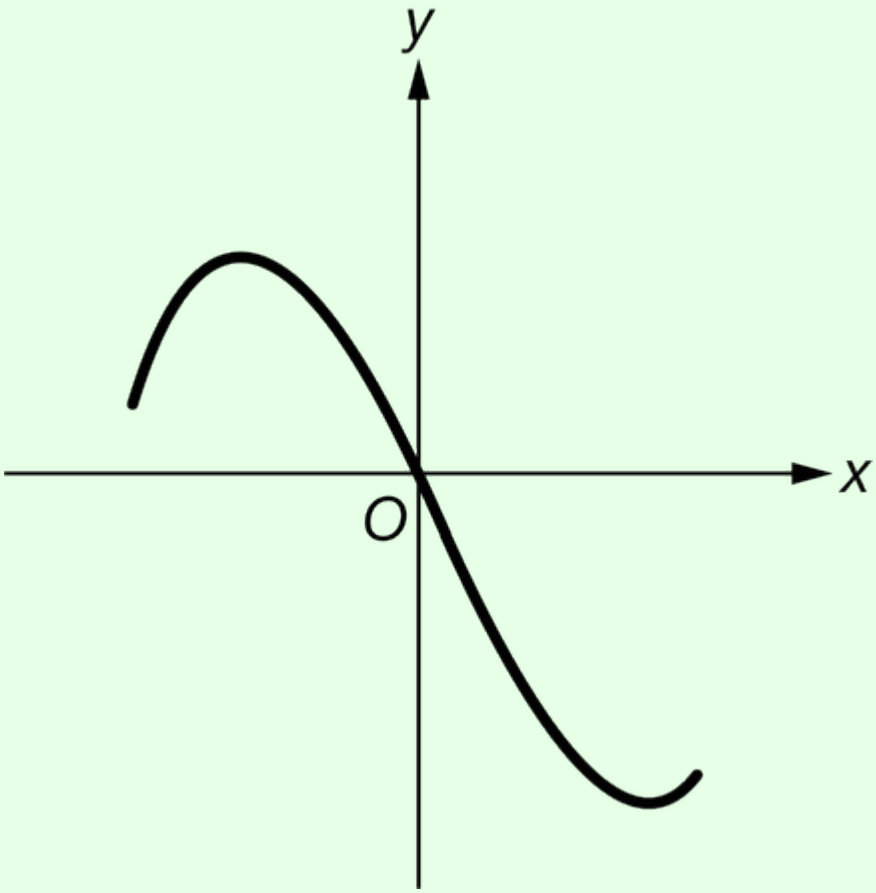
MC 5.6-5.9 Connections among f, f', f'' and graphs

(B)



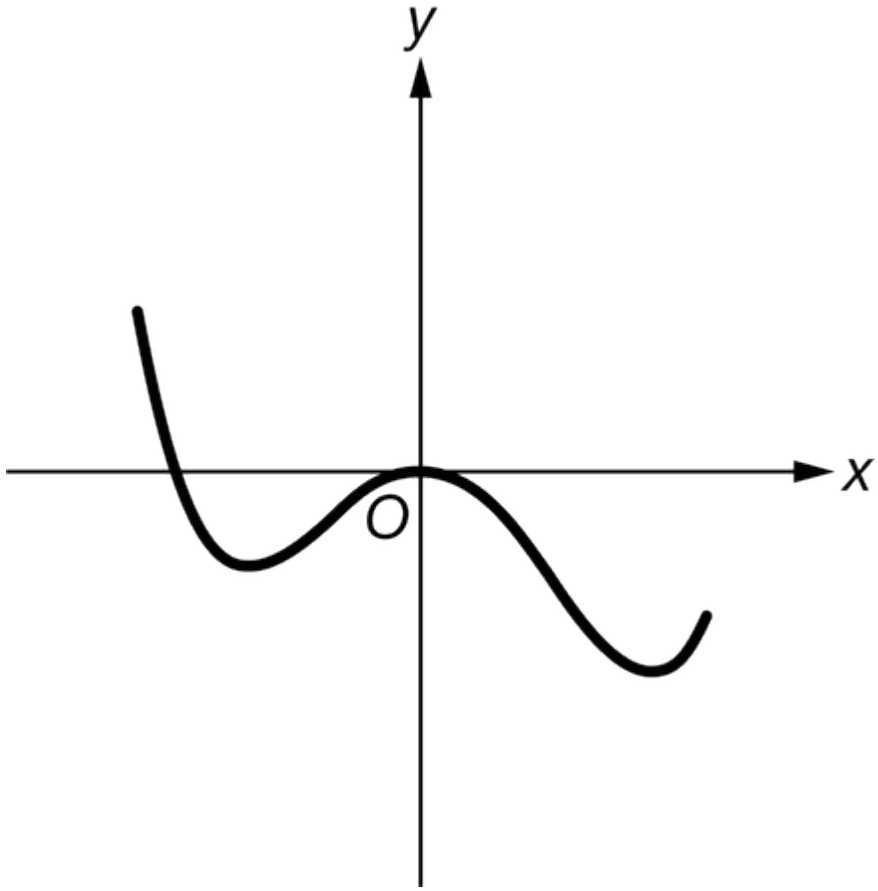
MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)



MC 5.6-5.9 Connections among f, f', f'' and graphs

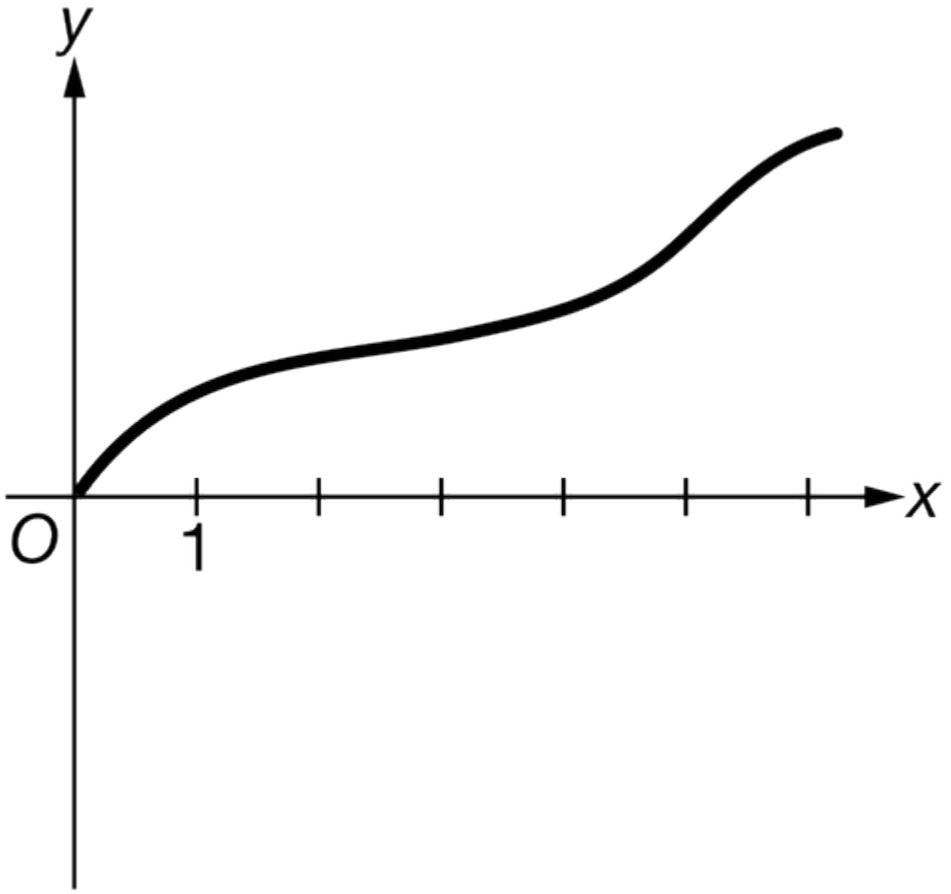
(D)



10. The function f is differentiable and increasing on the interval $0 \leq x \leq 6$, and the graph of f has exactly two points of inflection on this interval. Which of the following could be the graph of f', the derivative of f ?

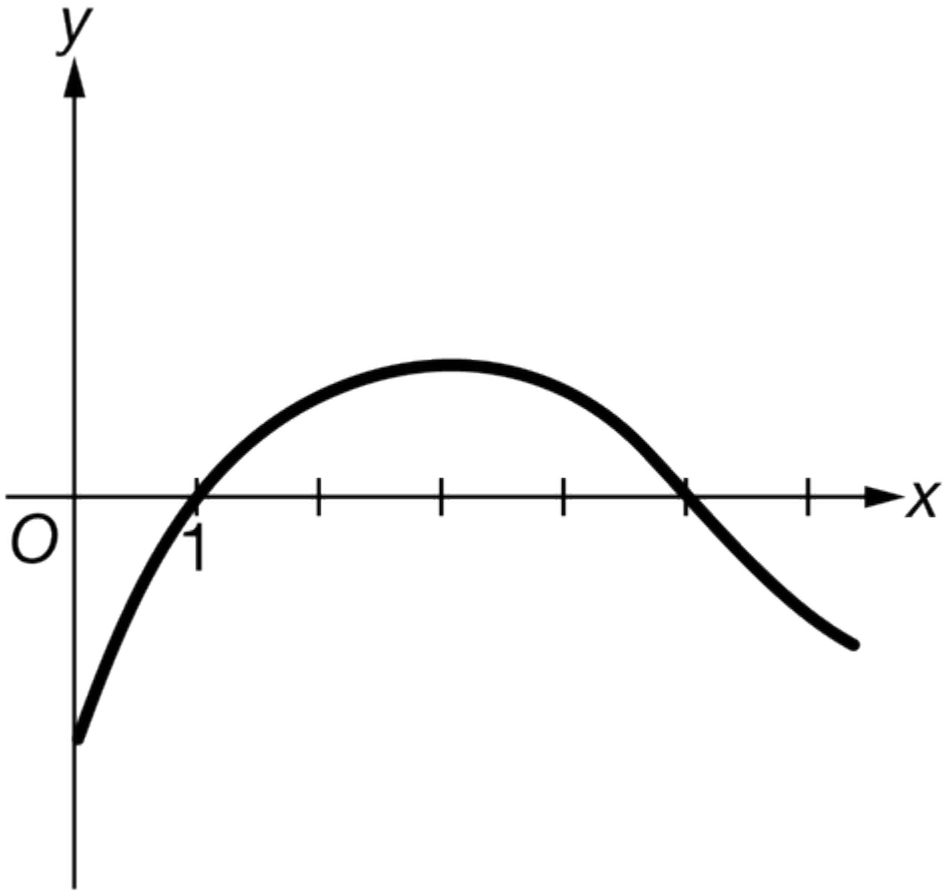
MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)



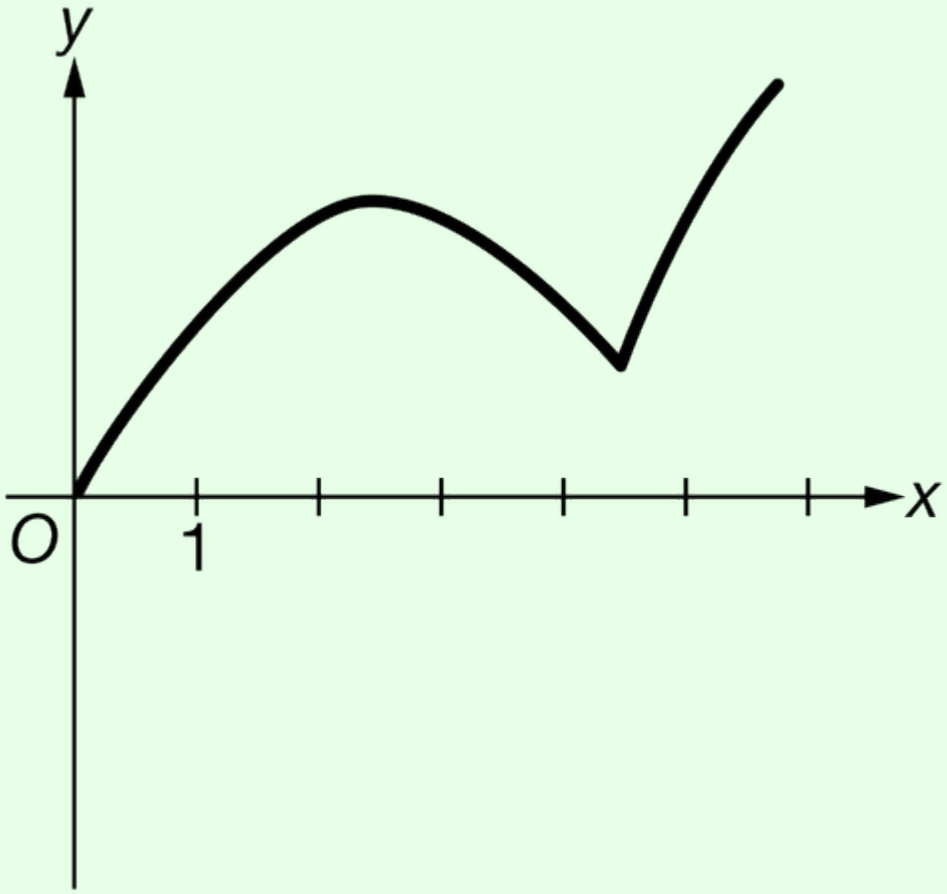
MC 5.6-5.9 Connections among f, f', f'' and graphs

(B)



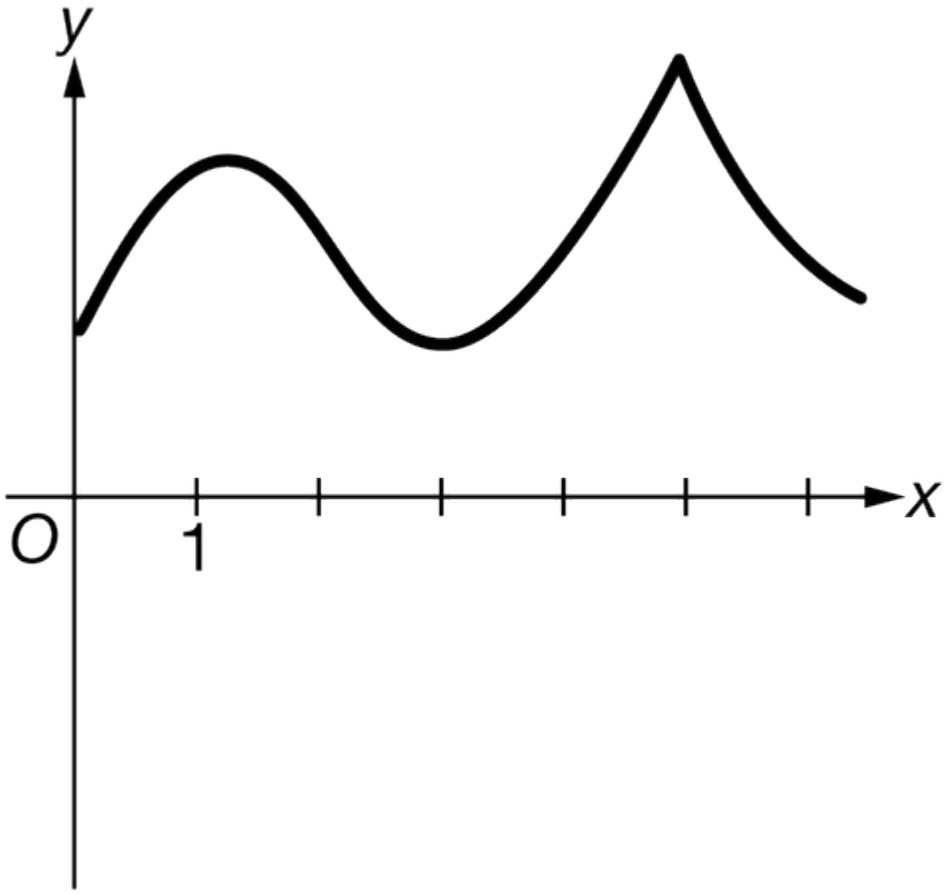
MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)



MC 5.6-5.9 Connections among f, f', f'' and graphs

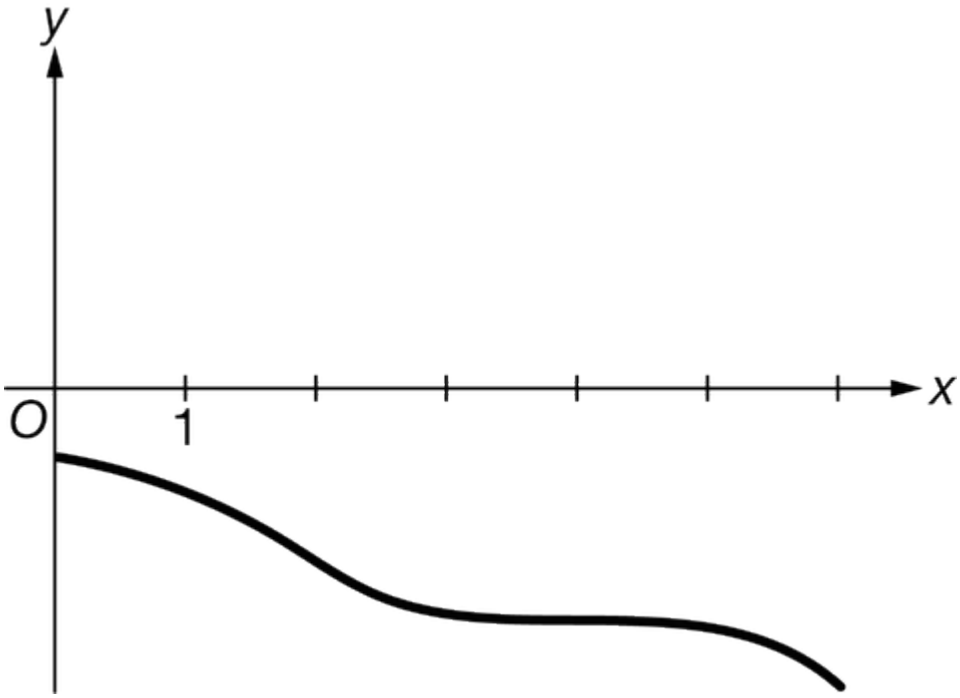
(D)



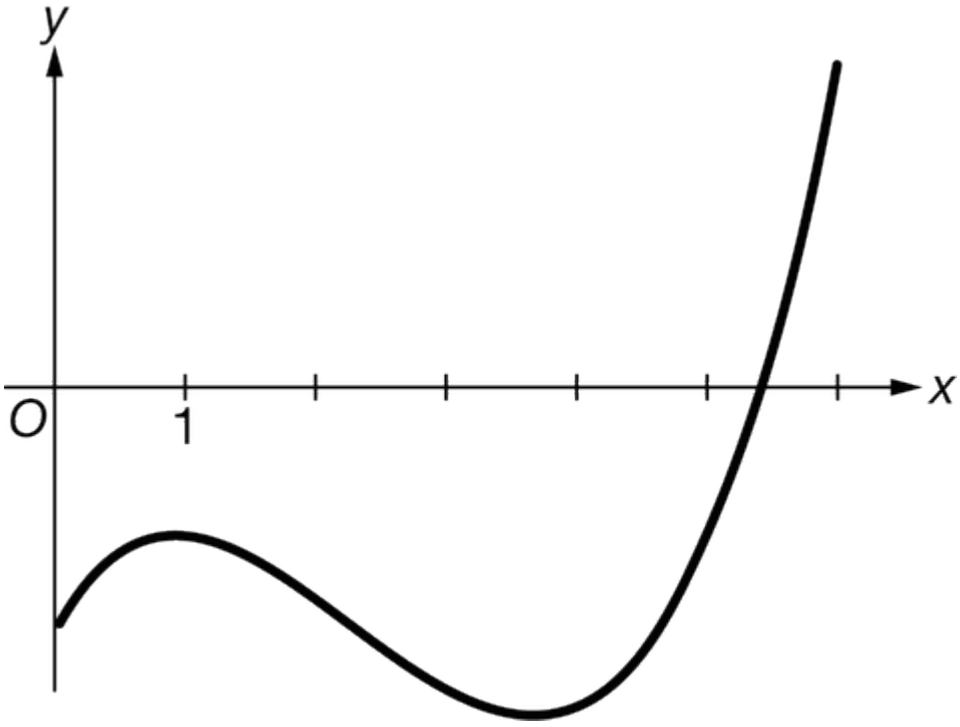
11. The function f is differentiable and decreasing on the interval $0 \leq x \leq 6$, and the graph of f has exactly two points of inflection on this interval. Which of the following could be the graph of f', the derivative of f ?

MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)

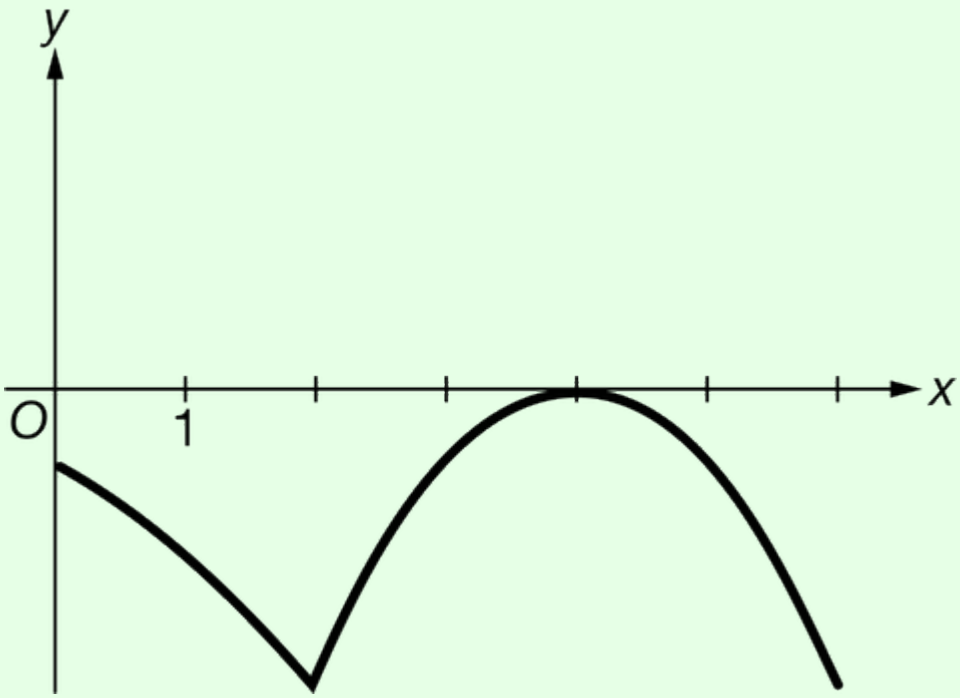


(B)

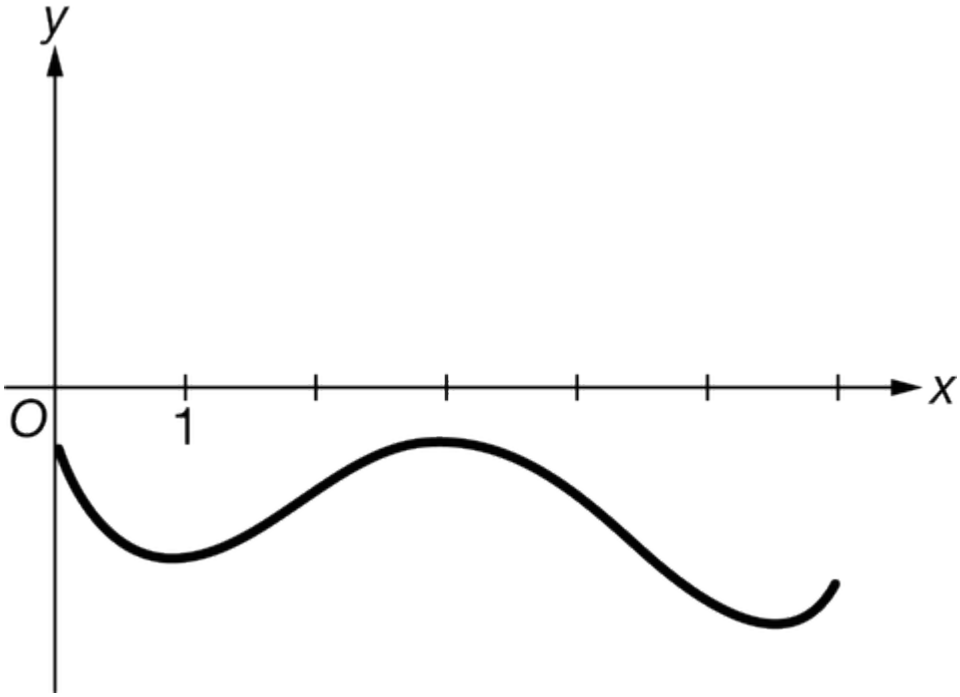


MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)

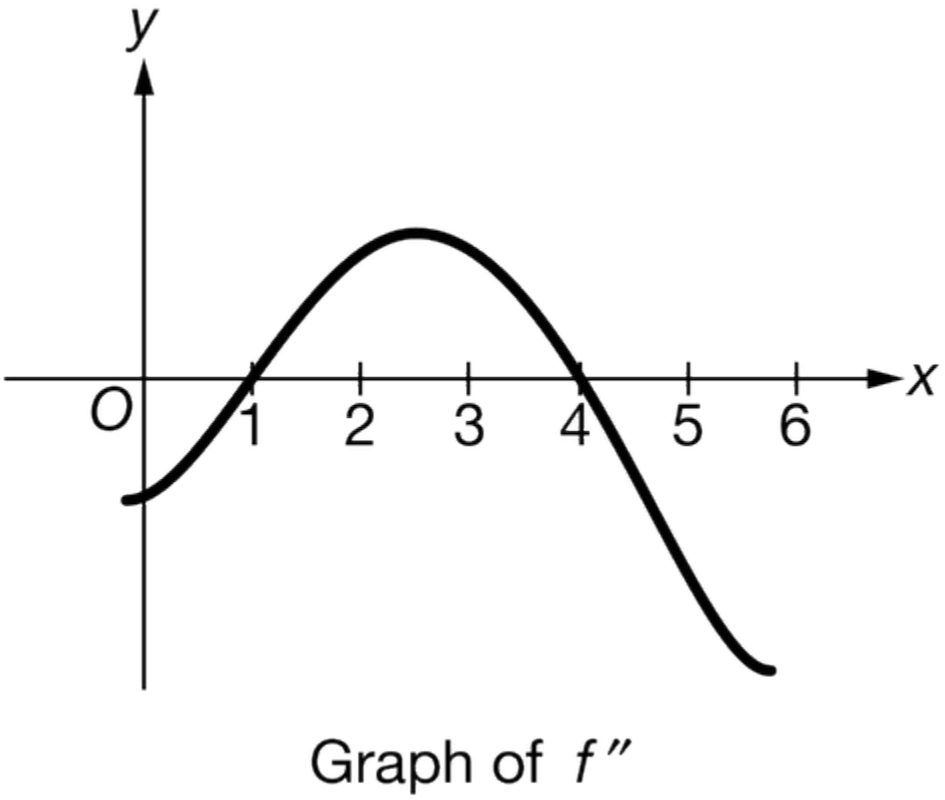


(D)



MC 5.6-5.9 Connections among f, f', f'' and graphs

12.

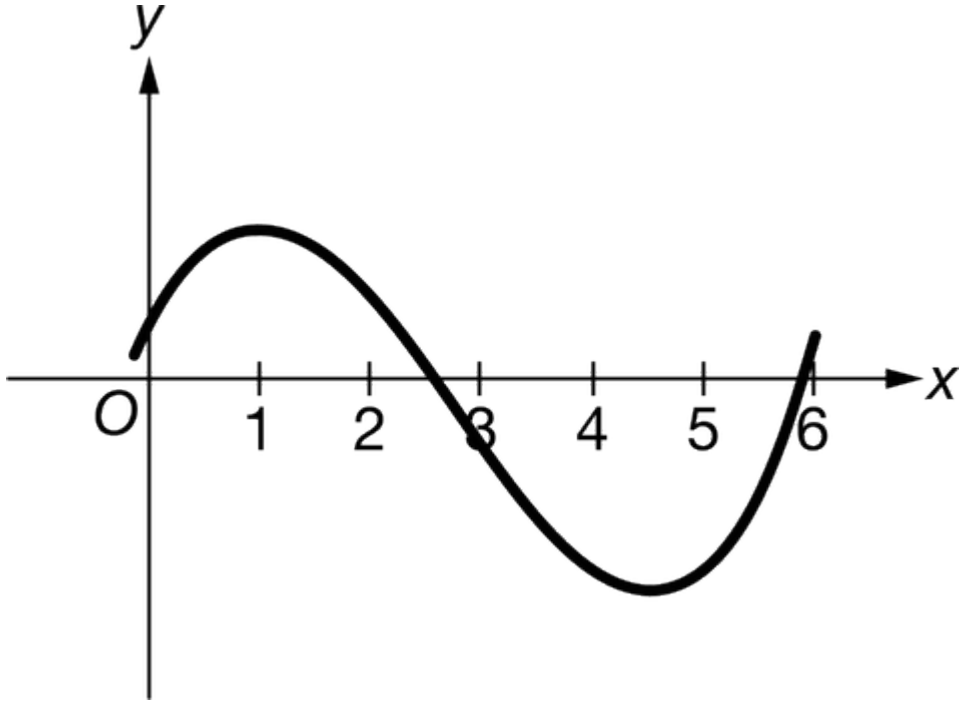


Graph of f''

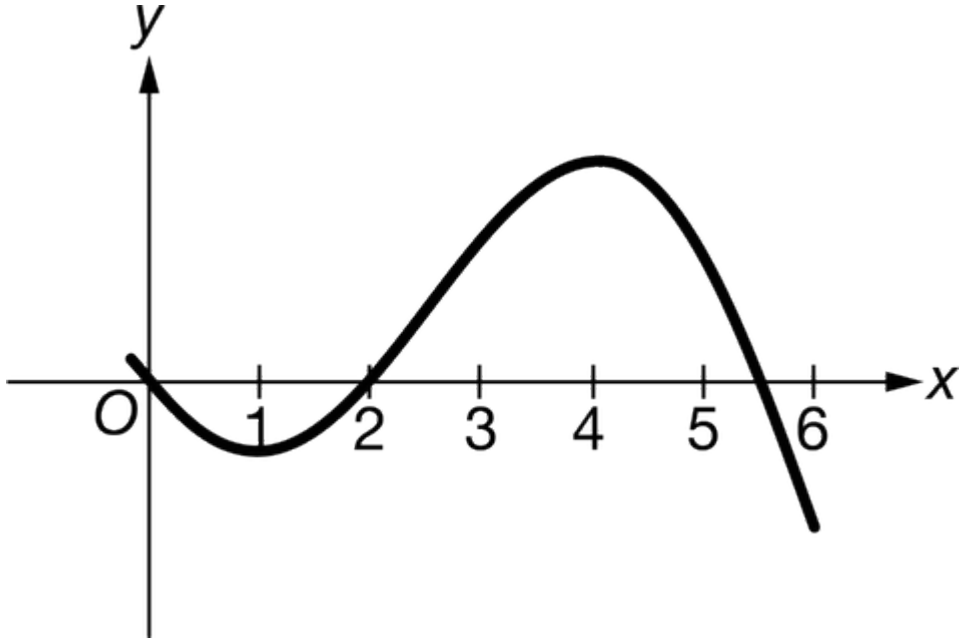
The graph of f'', the second derivative of the function f, is shown above on the interval $0 \leq x \leq 6$. Which of the following could be the graph of f ?

MC 5.6-5.9 Connections among f, f', f'' and graphs

(A)

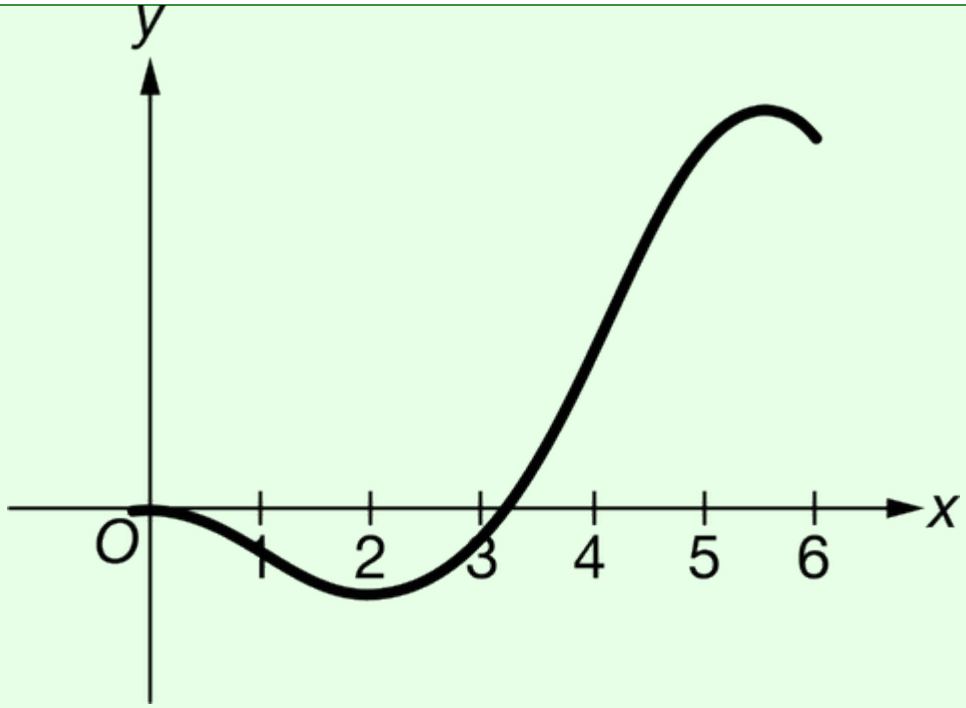


(B)

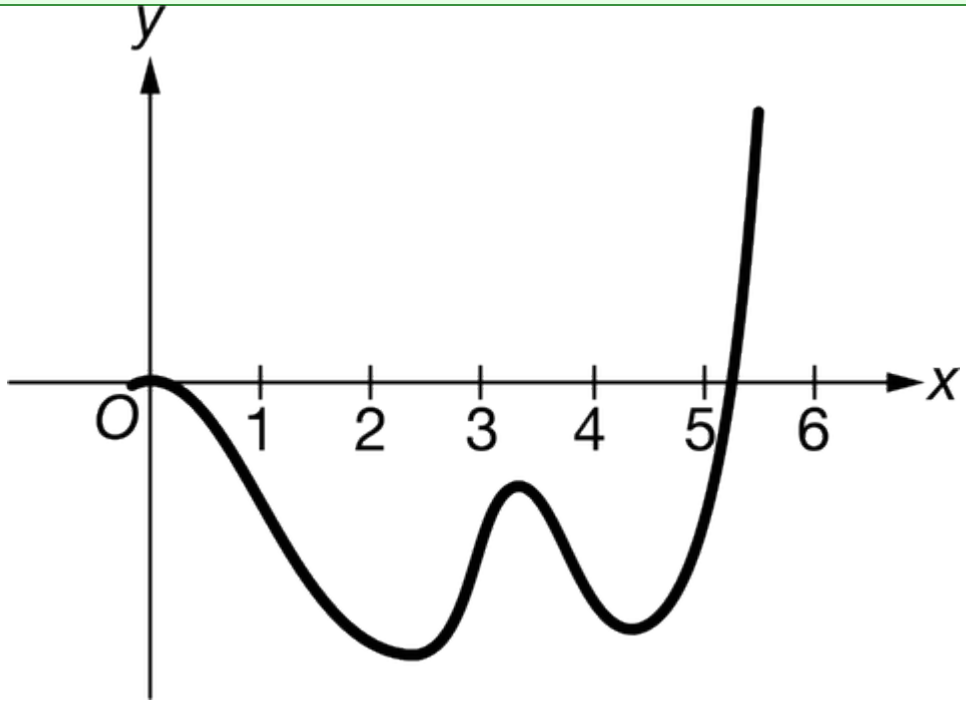


MC 5.6-5.9 Connections among f, f', f'' and graphs

(C)



(D)



13. The first derivative of the function h is given by $h'(x) = x^4 - x^3 + x$. On which of the following intervals is the graph of h concave down?

(A) $(-0.755, 0)$

(B) $(0, 0.5)$ only

(C) $(-0.455, \infty)$

(D) $(-\infty, -0.455)$

MC 5.6-5.9 Connections among f, f', f'' and graphs

14. The second derivative of the function g is given by $g''(x) = x^5 - 2.2x^4 - 6.61x^3 + 8.602x^2$. At which values of x in the interval $-3 < x < 4$ does the graph of g have a point of inflection where the concavity of the graph changes from concave up to concave down?

(A) $x = 1.1$ only ✓

(B) $x = -2.3$ and $x = 3.4$ only

(C) $x = -2.3$, $x = 1.1$, and $x = 3.4$ only

(D) $x = -2.3$, $x = 0$, $x = 1.1$, and $x = 3.4$

15. At what values of x does the graph of $y = e^{-x} + 2xe^{-x} + x^2e^{-x}$ have a point of inflection?

(A) $x = -1$ only

(B) $x = -1$ and $x = 1$

(C) $x = -3 - \sqrt{2}$ and $x = -3 + \sqrt{2}$

(D) $x = 1 - \sqrt{2}$ and $x = 1 + \sqrt{2}$ ✓

16. The first derivative of the function h is given by $h'(x) = x^5 - 3x^2 + x$. What are all intervals on which the graph of h is concave down?

(A) $(-\infty, 0)$ and $(0.338, 1.307)$

(B) $(-\infty, 0.669)$

(C) $(-\infty, 0.167)$ and $(1, \infty)$

(D) $(0.167, 1)$ ✓

17. At what values of x does the graph of $y = x^2e^{-2x}$ have a point of inflection?

(A) $x = -2$ and $x = 0$

(B) $x = 0$ and $x = 1$

(C) $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$

(D) $x = 1 - \frac{\sqrt{2}}{2}$ and $x = 1 + \frac{\sqrt{2}}{2}$ ✓

18. The second derivative of the function g is given by $g''(x) = 0.1x^5 - 0.29x^4 - 0.694x^3 + 1.9136x^2$. At which values of x in the interval $-3 < x < 4$ does the graph of g have a point of inflection where the concavity of the graph changes from concave up to concave down?

(A) $x = 2.3$ only ✓

(B) $x = -2.6$ and $x = 3.2$ only

(C) $x = -2.6$, $x = 2.3$, and $x = 3.2$ only

(D) $x = -2.6$, $x = 0$, $x = 2.3$, and $x = 3.2$

MC 5.6-5.9 Connections among f, f', f'' and graphs

19.

x	$0 < x < 5$	$x = 5$	$5 < x < 8$	$x = 8$	$8 < x < 12$	$x = 12$	$12 < x < 16$
$f'(x)$	Positive	Undefined	Negative	-2	Negative	0	Positive
$f''(x)$	Positive	Undefined	Negative	0	Positive	0	Positive

The function f is continuous on the interval $(0, 16)$, and f is twice differentiable except at $x = 5$ where the derivatives are undefined. Information about the first and second derivatives of f for values of x in the interval $(0, 16)$ is given in the table above. At what values of x in the interval $(0, 16)$ does the graph of f have a point of inflection?

(A) $x = 8$ only

(B) $x = 5$ and $x = 8$ ✓

(C) $x = 5$ and $x = 12$

(D) $x = 8$ and $x = 12$

20. Let f be the function defined by $f(x) = \frac{1}{3}x^3 - 3x^2 - 16x$. On which of the following intervals is the graph of f both decreasing and concave down?

(A) $(-\infty, 3)$

(B) $(-2, 3)$ only ✓

(C) $(3, 8)$

(D) $(8, \infty)$

21. Let f be the function defined by $f(x) = \frac{1}{3}x^3 - 4x^2 - 9x + 5$. On which of the following intervals is the graph of f both decreasing and concave down?

(A) $(-\infty, 4)$

(B) $(-1, 4)$ ✓

(C) $(4, 9)$

(D) $(9, \infty)$

22. The first derivative of the function h is given by $h'(x) = 3\ln(2 + \cos(2x)) - x$, and the second derivative of h is given by $h''(x) = \frac{-6\sin(2x)}{2+\cos(2x)} - 1$. On what open intervals contained in $-2 < x < 2$ is the graph of h both increasing and concave down?

(A) $(-2, -1.486)$ and $(-0.250, 1.085)$ ✓

(B) $(-2, -1.486)$ and $(-0.250, 1.656)$

(C) $(-2, 1.085)$

(D) $(-1.047, -0.250)$

MC 5.6-5.9 Connections among f, f', f'' and graphs

23. The first derivative of the function h is given by $h'(x) = \sin x + \cos\left(x^2\right) + x$, and the second derivative of h is given by $h''(x) = \cos x - 2x \sin\left(x^2\right) + 1$. On what open intervals contained in $-3 < x < 2$ is the graph of h both increasing and concave down?

(A) $(0.969, 1.697)$ only ✓

(B) $(-2.499, -1.829)$ and $(0.969, 1.697)$

(C) $(-0.495, 2)$

(D) $(-1.311, -0.166)$

24.

x	$0 < x < 3$	$x = 3$	$3 < x < 9$	$x = 9$	$9 < x < 11$	$x = 11$	$11 < x < 16$
$f'(x)$	Positive	Undefined	Negative	-3	Negative	0	Positive
$f''(x)$	Positive	Undefined	Negative	0	Positive	0	Positive

The function f is continuous on the interval $(0, 16)$, and f is twice differentiable except at $x = 3$, where the derivatives are undefined. Information about the first and second derivatives of f for values of x in the interval $(0, 16)$ is given in the table above. At what values of x in the interval $(0, 16)$ does the graph of f have a point of inflection?

(A) $x = 9$ only

(B) $x = 3$ and $x = 9$ ✓

(C) $x = 3$ and $x = 11$

(D) $x = 9$ and $x = 11$