Unit 1 Progress Check: MCQ Part B

1. If f is the function defined by $f(x)=\frac{x-1}{1-\frac{1}{x}}$, then $\lim_{x\to 1} f(x)$ is equivalent to which of the following?

(A) $\lim_{x\to 1} x$ ✓

(B) $\lim_{x\to 1} 2x$

(C) $\lim_{x\to 1}\left(\frac{x-1}{1-x}\right)$

(D) $\frac{\lim_{x\to 1}(x-1)}{\lim_{x\to 1}\left(1-\frac{1}{x}\right)}$

2. Let f and g be functions such that $\lim_{x\to 4} g(x)=7$ and $\lim_{x\to 4}\frac{f(x)}{g(x)}=\pi$. What is $\lim_{x\to 4} f(x)$?

(A) $\frac{\pi}{7}$

(B) $7+\pi$

(C) 7π ✓

(D) The limit cannot be determined from the information given.

3.

$$f(x)=\begin{cases}\frac{|x|}{x} & \text{for } x\neq 0\\ 0 & \text{for } x=0\end{cases}$$

If f is the function defined above, then $\lim_{x\to 0} f(x)$ is

(A) -1

(B) 0

(C) 1

(D) nonexistent ✓

4. The function f is defined for all x in the interval $3<x<6$. Which of the following statements, if true, implies that $\lim_{x\to 5} f(x)=12$?

(A) There exists a function g with $f(x)\leq g(x)$ for $3<x<6$, and $\lim_{x\to 5} g(x)=12$.

(B) There exists a function g with $g(x)\leq f(x)$ for $3<x<6$, and $\lim_{x\to 5} g(x)=12$.

(C) There exist functions g and h with $g(x)\leq f(x)\leq h(x)$ for $3<x<6$, and $\lim_{x\to 5} g(x)=11$ and $\lim_{x\to 5} h(x)=13$.

(D) There exist functions g and h with $g(x)\leq f(x)\leq h(x)$ for $3<x<6$, and $\lim_{x\to 5} g(x)=\lim_{x\to 5} h(x)=12$. ✓

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5. The function g is given by $g(x)=\frac{1}{x^2-4x+5}$. The function h is given by $h(x)=\frac{2x^2-8x+10}{x^2-4x+6}$. If f is a function that satisfies $g(x)\le f(x)\le h(x)$ for $0<x<5$, what is $\lim\limits_{x\to 2} f(x)$?

(A) 0

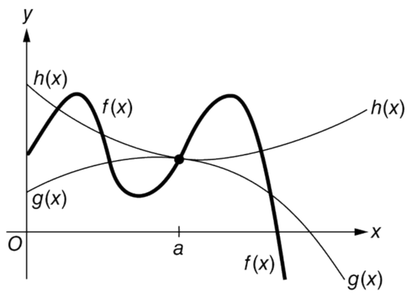
(B) 1

(C) 2

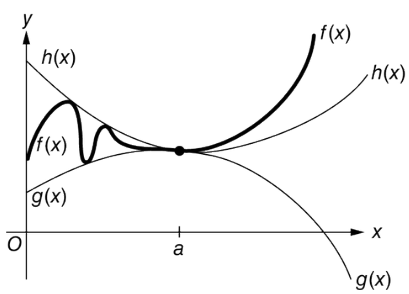
(D) The limit cannot be determined from the information given.

6. Let f be a function of x. The value of $\lim\limits_{x\to a} f(x)$ can be found using the squeeze theorem with the functions g and h. Which of the following could be graphs of f, g, and h ?

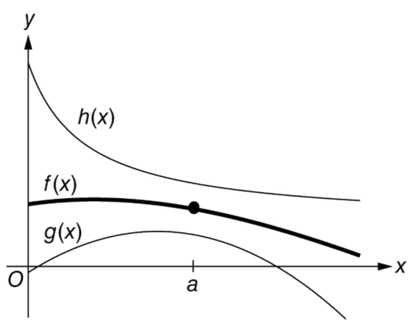
(A)



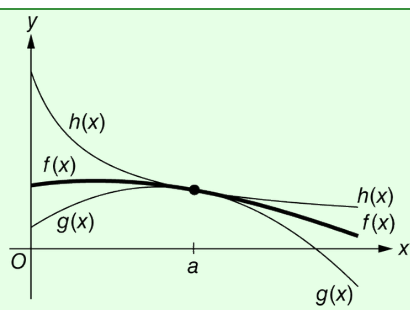
(B)



(C)



(D)



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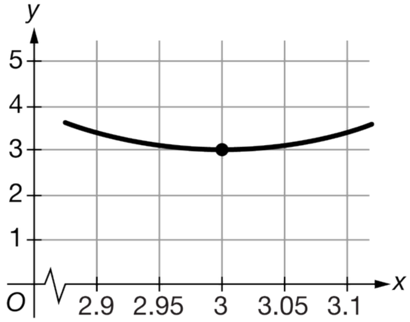
7.

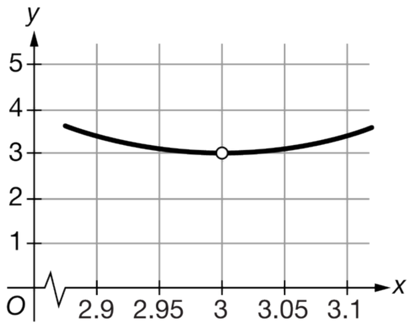
x	2.9	2.95	2.99	2.999	3.001	3.01	3.05	3.1
$f(x)$	3.4	3.1	3.004	3.00004	3.00004	3.004	3.1	3.4

The table above gives selected values for a function f. Based on the data in the table, which of the following could not be the graph of f on the interval $2.9 \leq x \leq 3.1$?

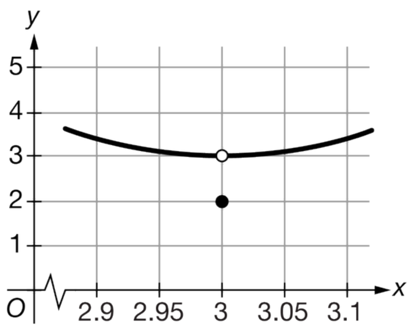
(A)



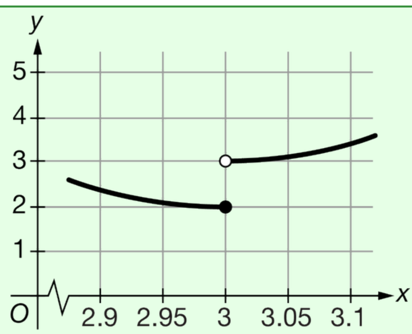
(B)



(C)



(D)



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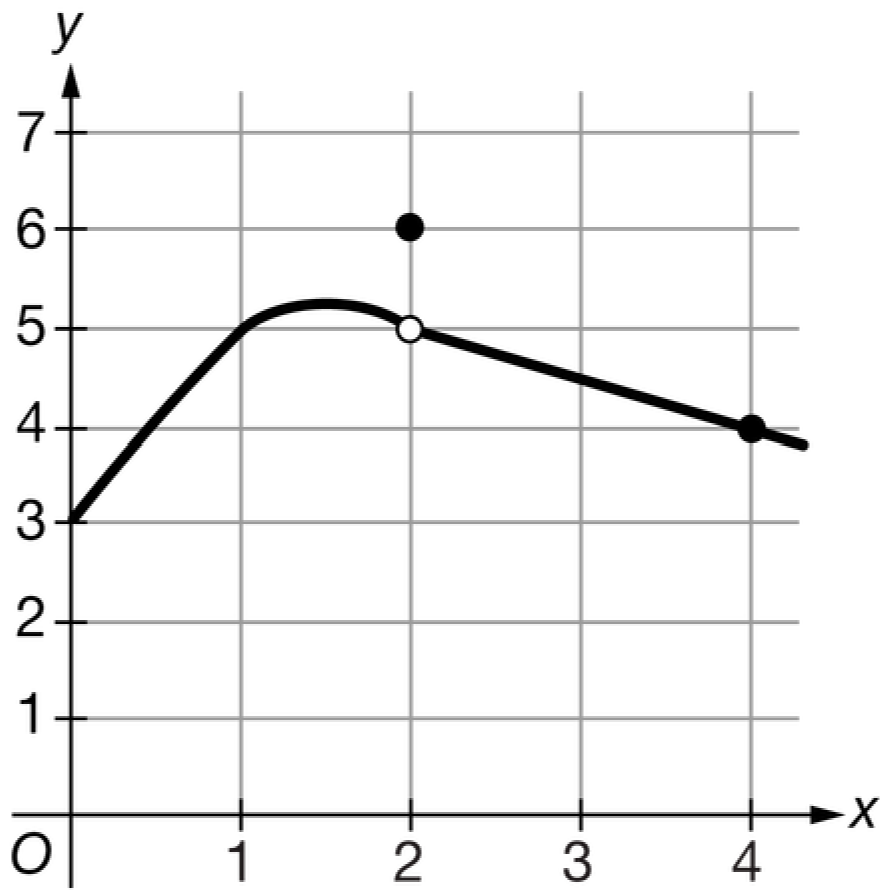
8. $$f(x)=\begin{cases} -x^2+3x+3 & \text{for } x<2 \\ 6 & \text{for } x=2 \\ 6-\frac{1}{2}x & \text{for } x>2 \end{cases}$$



Let f be the piecewise function defined above. Also shown is a portion of the graph of f. What is the value of $\lim_{x\to 2} f(f(x))$?

(A) -15

(B) -7

(C) 3

(D) $\frac{7}{2}$ ✓

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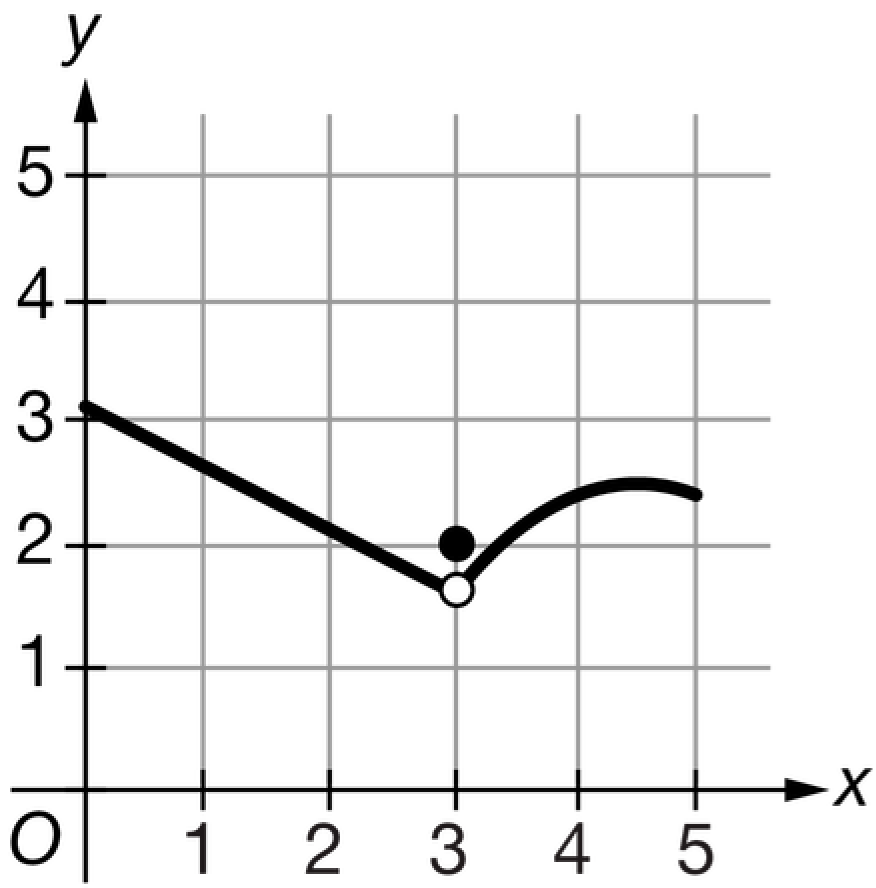
9.

x	2.9	2.95	2.99	2.998	3.002	3.01	3.05	3.1
$f(x)$	1.650	1.625	1.605	1.601	1.602	1.612	1.659	1.716



The table above gives selected values for a function f. Also shown is a portion of the graph of f. The graph consists of a line segment for $x < 3$ and part of a parabola for $x > 3$. What is $\lim_{x \to 3} f(x)$?

(A) 1.6 ✓

(B) $\frac{1.601+1.602}{2}$

(C) 2

(D) The limit does not exist.

10. $$f(x) = \begin{cases} \frac{2x^2-5x-3}{x-3} & \text{if } x \neq 3 \\ 9 & \text{if } x = 3 \end{cases}$$

The function f is defined above. Which of the following statements is true?

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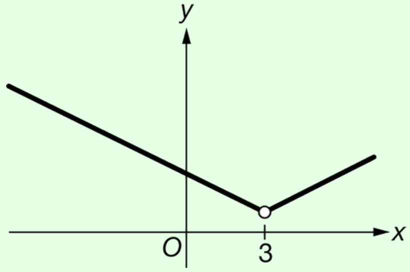
(A) f is continuous at $x = 3$.

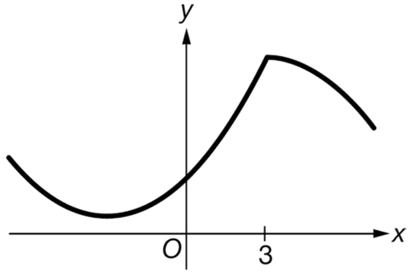
(B) f has a removable discontinuity at $x = 3$.

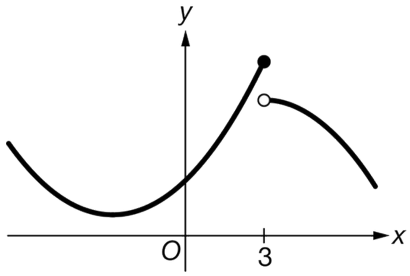
(C) f has a jump discontinuity at $x = 3$.

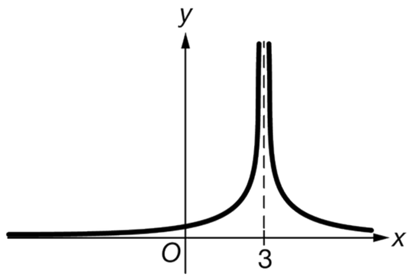
(D) f has a discontinuity due to a vertical asymptote at $x = 3$.

11. The function f has a removable discontinuity at $x = 3$. Which of the following could be the graph of f ?

(A) 

(B) 

(C) 

(D) 

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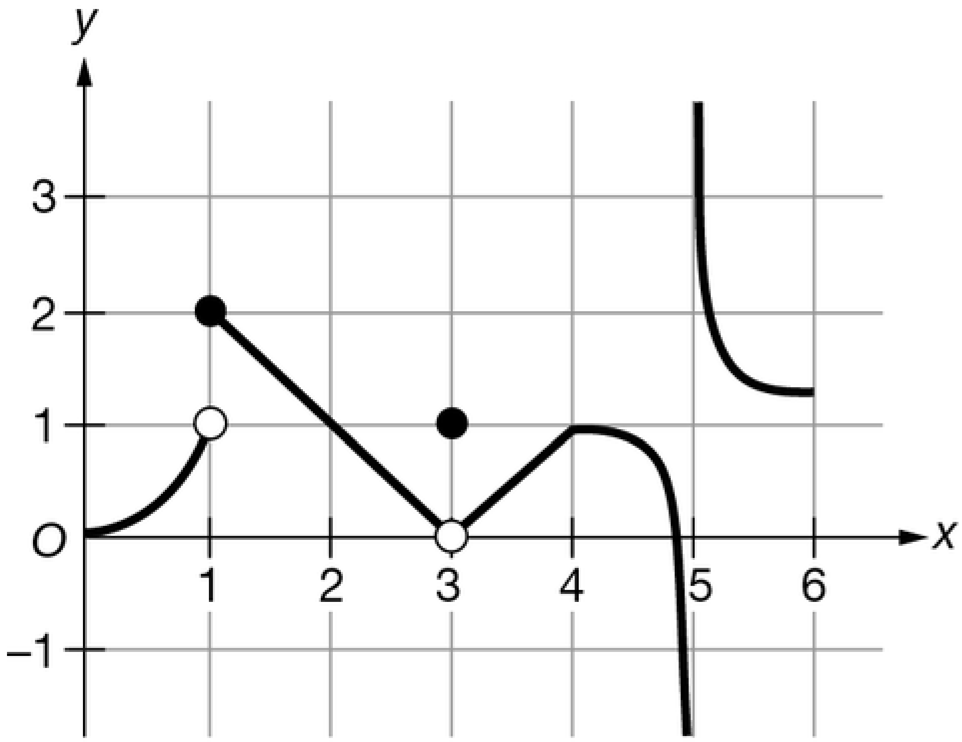
12.



Graph of f

The graph of a function f is shown in the figure above. At what value of x does f have a jump discontinuity?

(A) $x = 1$ ✓

(B) $x = 3$

(C) $x = 4$

(D) $x = 5$

13. If $\lim_{x\to 6} f(x)$ exists with $\lim_{x\to 6} f(x) < 8$ and $f(6) = 12$, which of the following statements must be false?

(A) $\lim_{x\to 6^-} f(x) = 0$

(B) $\lim_{x\to 6^+} f(x) < 8$

(C) $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$

(D) f is continuous at $x = 6$. ✓

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14. $$f(x) = \begin{cases} 3^x & \text{for } 0 < x < 1 \\ \frac{1}{2}x^2 - x + \frac{7}{2} & \text{for } 1 < x < 2 \end{cases}$$

Let f be the function defined above. Which of the following statements is true?

(A) f is continuous at $x = 1$.

(B) f is not continuous at $x = 1$ because $f(1)$ does not exist.

(C) f is not continuous at $x = 1$ because $\lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$.

(D) f is not continuous at $x = 1$ because $\lim\limits_{x \to 1} f(x)$ does not exist.

15. Which of the following functions is continuous at $x = 3$?

(A) $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3} & \text{for } x \neq 3 \\ 8 & \text{for } x = 3 \end{cases}$

(B) $g(x) = \begin{cases} 5 & \text{for } x < 3 \\ 3x - 4 & \text{for } x > 3 \end{cases}$

(C) $h(x) = \begin{cases} \sin\left(\frac{\pi}{2}x\right) & \text{for } x < 3 \\ -1 & \text{for } x = 3 \\ \cos(\pi x) & \text{for } x > 3 \end{cases}$

(D) $k(x) = \begin{cases} 5 + \ln(4 - x) & \text{for } x \leq 3 \\ 5\ln(x - 2) & \text{for } x > 3 \end{cases}$