Unit 3 Progress Check: MCQ

1. If $g(x) = 2\ln(x+1)$ and f is a differentiable function of x, which of the following is equivalent to the derivative of $f(g(x))$ with respect to x ?

(A) $f'\left(\frac{2}{x+1}\right)$

(B) $\frac{2f'(x)}{x+1}$

(C) $f'(2\ln(x+1))$

(D) $\frac{2f'(2\ln(x+1))}{x+1}$ ✓

2. For which of the following functions is the chain rule an appropriate method to find the derivative with respect to x ?

I. $y = \cos\left(\sqrt{x}+1\right)$
II. $y = 2^x \sin x$
III. $y = \frac{20}{40x^2-1}$

(A) I only

(B) II only

(C) III only

(D) I and III only ✓

3. Let f be a differentiable function. If $h(x) = (2+f(\sin x))^3$, which of the following gives a correct process for finding $h'(x)$?

(A) $h'(x) = 3(2+f(\sin x))^2$

(B) $h'(x) = 3(2+f(\sin x))^2 \cdot f'(\sin x)$

(C) $h'(x) = 3(2+f(\sin x))^2 \cdot f'(\cos x)$

(D) $h'(x) = 3(2+f(\sin x))^2 \cdot f'(\sin x) \cdot \cos x$ ✓

4. What is the slope of the line tangent to the curve $4y^2 + xy - 2x^2 = 3$ at the point $(-1,-1)$?

(A) -5

(B) $-\frac{3}{7}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$ ✓

5. If $\cos(4x-y) = x+y$, then $\frac{dy}{dx} =$

Unit 3 Progress Check: MCQ

(A) $-1-\sin(4x-y)$

(B) $\frac{2+4\sin(4x-y)}{\sin(4x-y)}$

(C) $-\frac{1}{1+\sin(4x-y)}$

(D) $\frac{1+4\sin(4x-y)}{-1+\sin(4x-y)}$ ✓

6.

$f(1)=4$	$f'(1)=-2$	$g(3)=7$	$g'(3)=1$

The point $(1,3)$ lies on the curve in the xy-plane given by the equation $f(x)g(y)=24+x+y$, where f is a differentiable function of x and g is a differentiable function of y. Selected values of f, f', g, and g' are given in the table above. What is the value of $\frac{dy}{dx}$ at the point $(1,3)$?

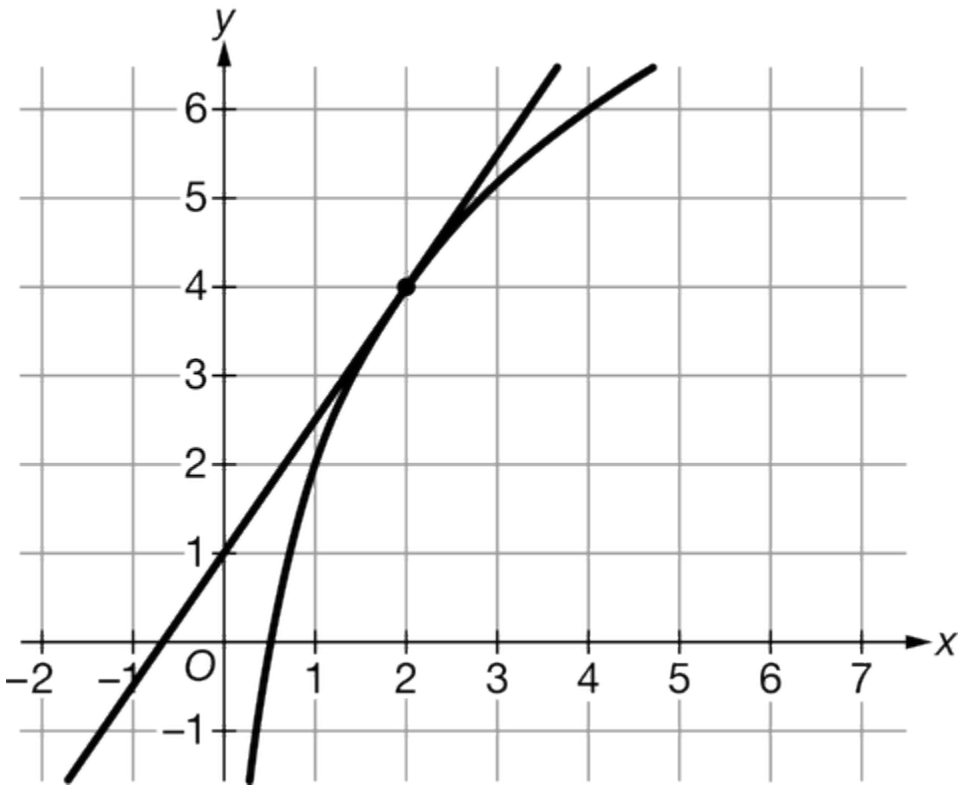
(A) -11

(B) 4

(C) 5 ✓

(D) 13

7.



The graph of the increasing differentiable function f is shown above. Also shown is the line tangent to the graph of f at the point $(2,4)$. Let g be the inverse of f. Which of the following statements about g' is true?

Unit 3 Progress Check: MCQ

(A) $g'(2) = \frac{2}{3}$

(B) $g'(2) = \frac{3}{2}$

(C) $g'(4) = \frac{2}{3}$ ✓

(D) $g'(4) = \frac{3}{2}$

8. Let f be the decreasing function defined by $f(x) = -x^3 - 6x^2 - 12x + 8$, where $f(4) = -8$. If g is the inverse function of f, which of the following is a correct expression for $g'(-8)$?

(A) $g'(-8) = \frac{1}{f'(-8)}$

(B) $g'(-8) = \frac{1}{f'(4)}$ ✓

(C) $g'(-8) = f'(4)$

(D) $g'(-8) = f'(-8)$

9.

x	-4	0	3
$f(x)$	0	3	5
$f'(x)$	1	2	4

The table above gives selected values for a differentiable and increasing function f and its derivative. If $g(x) = f^{-1}(x)$ for all x, which of the following is a correct expression for $g'(0)$?

(A) $g'(0) = f'(0) = 2$

(B) $g'(0) = \frac{1}{f'(0)} = \frac{1}{2}$

(C) $g'(0) = \frac{1}{f'(-4)} = 1$ ✓

(D) $g'(0) = -\frac{f'(0)}{(f(0))^2} = -\frac{2}{9}$

10. $\frac{d}{dx}\left(\sin^{-1}\left(x^2\right)\right)\Big|_{x=\frac{1}{4}} =$

(A) $\frac{2\left(\frac{1}{4}\right)}{1+\left(\frac{1}{4}\right)^4}$

(B) $\frac{2\left(\frac{1}{4}\right)}{\sqrt{1-\left(\frac{1}{4}\right)^4}}$ ✓

(C) $2\left(\frac{1}{4}\right)\cos^{-1}\left(\frac{1}{16}\right)$

(D) $-2\left(\frac{1}{4}\right)\cot\left(\frac{1}{16}\right)\csc\left(\frac{1}{16}\right)$

11. $\frac{d}{dx}\left(\cos^{-1}(-3x)\right) =$

Unit 3 Progress Check: MCQ

(A) $\frac{3}{\sqrt{1-(-3x)^2}}$ ✓

(B) $\frac{-3}{\sqrt{1-(-3x)^2}}$

(C) $-\sin^{-1}(-3x)\cdot(-3)$

(D) $-\cos^{-2}(-3x)\cdot(-3)$

12. Which of the following methods can be used to find the derivative of $y=\arccos\left(\sqrt{x}\right)$ with respect to x ?

I. Use the quotient rule to differentiate $\frac{1}{\cos\left(\sqrt{x}\right)}$.
II. Use the chain rule to differentiate $\cos\left(\arccos\left(\sqrt{x}\right)\right)=\sqrt{x}$.
III. Use implicit differentiation to differentiate the function y in the relation $\cos y=\sqrt{x}$ with respect to x.

(A) I only

(B) III only

(C) II and III only ✓

(D) I, II, and III

13. Which of the following expressions can be differentiated using the product rule?

(A) $\arcsin(\cos x)$

(B) $\sin x(\arccos x)$ ✓

(C) $e^x+\arctan x$

(D) $\left(12x^2+3x-6\right)^e$

14. Which of the following requires the use of implicit differentiation to find $\frac{dy}{dx}$?

(A) $2y+3x^2-x=5$

(B) $y=e^{3+x}+x^3$

(C) $y=e^{y+x}+x^3$ ✓

(D) $y=\frac{x^4+3}{4x^3-2}$

15. For which of the following functions would the quotient rule be considered the best method for finding the derivative?

(A) $y=\left(x^3+x\right)^{-2}$

(B) $y=\frac{x^3+x}{x}$

(C) $y=\cos^{-1}(x^3+x)$

(D) $y=\frac{\cos(x^3+x)}{x^3+x}$ ✓

16. If $y=3e^{-2x}$, then $\frac{d^3y}{dx^3}=$

Unit 3 Progress Check: MCQ

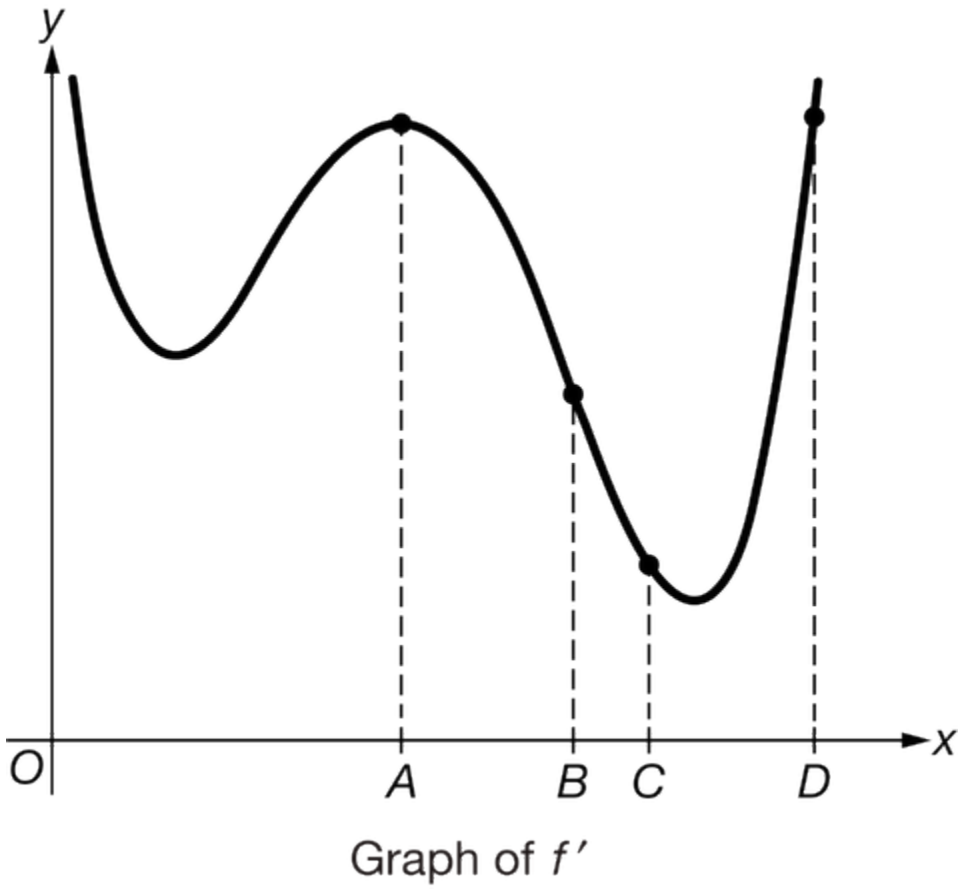
(A) $-24e^{-2x}$ ✓

(B) $-6e^{-2x}$

(C) $48e^{-2x}$

(D) $-216e^{-6x}$

17.



Graph of f'

The figure above shows the graph of f', the derivative of the function f. At which of the four indicated values of x is $f''(x)$ least?

(A) A

(B) B ✓

(C) C

(D) D

18. Let $y = f(x)$ be a twice-differentiable function such that $f(-1) = 5$ and $\frac{dy}{dx} = \frac{1}{5}\left(xy^2 + 4y\right)^2$. What is the value of $\frac{d^2y}{dx^2}$ at $x = -1$?

Unit 3 Progress Check: MCQ

(A) -190

(B) -70

(C) -2

(D) 10 ✓