Unit 5 Progress Check: MCQ Part A

1. Let f be the function given by $f(x) = \cos\left(x^2 + x\right) + 2$. The derivative of f is given by $f'(x) = -(2x + 1)\sin\left(x^2 + x\right)$. What value of c satisfies the conclusion of the Mean Value Theorem applied to f on the interval $[1, 2]$?

(A) 1.079 because $f(1.079) = \frac{f(2) - f(1)}{2 - 1}$

(B) 1.438 because $f'(1.438) = \frac{f(2) - f(1)}{2 - 1}$ ✓

(C) 1.750 because $f''(1.750) = 0$

(D) 1.932 because $f'(1.932) = f(1.932)$

2. The derivative of the function f is given by $f'(x) = \sqrt{x}\sin\left(3\sqrt{x}\right)$. On which of the following intervals in $[0, 6\pi]$ is f decreasing?

(A) $[0, 1.097]$, $[4.386, 9.870]$, $[17.546, 18.850]$

(B) $[0.457, 2.682]$, $[7.073, 13.654]$

(C) $[1.097, 4.386]$, $[9.870, 17.546]$ ✓

(D) $[0, 0.457]$, $[2.682, 7.073]$, $[13.654, 18.850]$

3. The concentration of a certain element in the water supply of a town is modeled by the function f, where $f(t)$ is measured in parts per billion and t is measured in years. The first derivative of f is given by $f'(t) = 1 - \ln t - \sin t$. At what times t, for $0 < t < 5$, does the concentration attain a local minimum?

(A) $t = 1.110$ only

(B) $t = 2.074$

(C) $t = 3.353$ only ✓

(D) $t = 1.110$ and $t = 3.353$

4. Let f be the function given by $f(x) = \frac{x^2 - 9}{\sin x}$ on the closed interval $[0, 5]$. Of the following intervals, on which can the Mean Value Theorem be applied to f ?

I. $[1, 3]$, because f is continuous on $[1, 3]$ and differentiable on $(1, 3)$.
II. $[4, 5]$, because f is continuous on $[4, 5]$ and differentiable on $(4, 5)$.
III. $[1, 4]$, because f is continuous on $[1, 4]$ and differentiable on $(1, 4)$.

(A) None

(B) I only

(C) I and II only ✓

(D) I, II, and III

5. Let f be a differentiable function with $f(-3) = 7$ and $f(3) = 8$. Which of the following must be true for some c in the interval $(-3, 3)$?

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(A) $f'(c) = 0$, since the Extreme Value Theorem applies.

(B) $f'(c) = \frac{8+7}{3-(-3)}$, since the Mean Value Theorem applies.

(C) $f'(c) = \frac{8-7}{3-(-3)}$, since the Mean Value Theorem applies. ✓

(D) $f'(c) = 7.5$, since the Intermediate Value Theorem applies.

6. Let f be the function given by $f(x) = \frac{\sin x \cos x}{(x^2-4)}$ on the closed interval $[-2\pi, 2\pi]$. On which of the following closed intervals is the function f guaranteed by the Extreme Value Theorem to have an absolute maximum and an absolute minimum?

(A) $[-2\pi, 2\pi]$

(B) $[-2, 2]$

(C) $[-1, 1]$ ✓

(D) $\left[\frac{\pi}{2}, \pi\right]$

7. Let f be the function defined by $f(x) = \frac{x^2+1}{x+1}$ with domain $[0, \infty)$. The function f has no absolute maximum on its domain. Why does this not contradict the Extreme Value Theorem?

(A) The domain of f is not an open interval.

(B) The domain of f is not a closed and bounded interval. ✓

(C) The function f is not continuous on its domain.

(D) The function f is not differentiable on its domain.

8.

x	0	1	2	3	4
$f(x)$	2	-2	4	7	18

Selected values of a continuous function f are given in the table above. Which of the following statements could be false?

(A) By the Intermediate Value Theorem applied to f on the interval $[0, 4]$, there is a value c such that $f(c) = 5$.

(B) By the Mean Value Theorem applied to f on the interval $[0, 4]$, there is a value c such that $f'(c) = 4$. ✓

(C) By the Extreme Value Theorem applied to f on the interval $[0, 4]$, there is a value c such that $f(c) \leq f(x)$ for all x in $[0, 4]$.

(D) By the Extreme Value Theorem applied to f on the interval $[0, 4]$, there is a value c such that $f(c) \geq f(x)$ for all x in $[0, 4]$.

9. Let f be the function defined by $f(x) = 3x^3 - 36x + 6$ for $-4 < x < 4$. Which of the following statements is true?

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(A) f is decreasing on the interval $(0, 4)$ because $f'(x) < 0$ on the interval $(0, 4)$.

(B) f is increasing on the interval $(0, 4)$ because $f'(x) < 0$ on the interval $(0, 4)$.

(C) f is decreasing on the interval $(-2, 0)$ because $f''(x) < 0$ on the interval $(-2, 0)$.

(D) f is decreasing on the interval $(-2, 2)$ because $f'(x) < 0$ on the interval $(-2, 2)$. ✓

10. Let f be the function defined by $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 6x$. On which open intervals is f decreasing?

(A) $-2 < x < 3$ only ✓

(B) $x < -2$ and $x > 3$

(C) $x < \frac{1}{2}$ only

(D) There are no such intervals.

11. Let f be a function with first derivative given by $f'(x) = (x + 1)(x - 2)(x - 3)$. At what values of x does f have a relative maximum?

(A) 2 only ✓

(B) -1 and 3 only

(C) 3 only

(D) -1, 2, and 3

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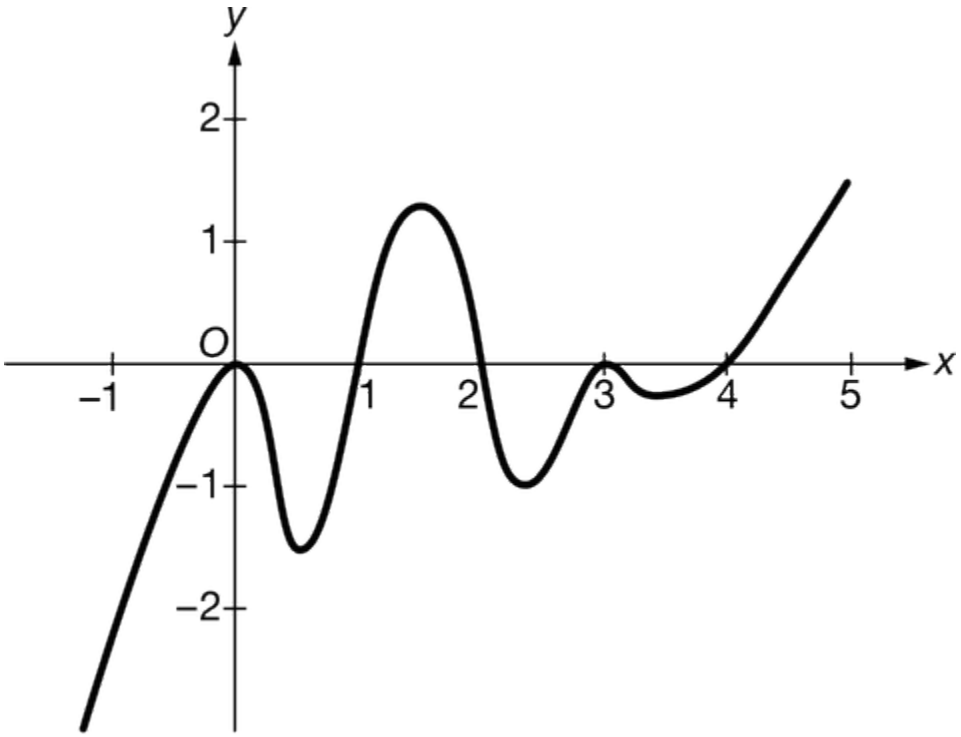
12.



Graph of f'

The graph of f', the derivative of the function f, is shown above for $-1 < x < 5$. Which of the following statements is true for $-1 < x < 5$?

(A) f has one relative minimum and two relative maxima.

(B) f has two relative minima and one relative maximum. ✓

(C) f has two relative minima and three relative maxima.

(D) f has three relative minima and three relative maxima.