Unit 5 Progress Check: MCQ Part B

1. The second derivative of the function f is given by $f''(x) = x^2 \cos\left(\sqrt{x}\right) - 2x \cos\left(\sqrt{x}\right) + \cos\left(\sqrt{x}\right)$. At what values of x in the interval $(0, 3)$ does the graph of f have a point of inflection?

(A) 2.467 only ✓

(B) 1 and 2.467

(C) 1.443 and 2.734

(D) 1 and 1.962

2. The second derivative of the function f is given by $f''(x) = e^{\sin x}\left(2\cos x - x\sin x + x\cos^2 x\right)$. The function f has many critical points, two of which are at $x = 2.074$ and $x = 7.980$. Which of the following statements is true?

(A) f has local minima at $x = 2.074$ and at $x = 7.980$.

(B) f has a local minimum at $x = 2.074$ and a local maximum at $x = 7.980$.

(C) f has a local maximum at $x = 2.074$ and a local minimum at $x = 7.980$.

(D) f has local maxima at $x = 2.074$ and at $x = 7.980$. ✓

3. Let f be the function given by $f(x) = -x^3 + 3x^2 + 24x$. What is the absolute maximum value of f on the closed interval $[-6, 6]$?

(A) -6

(B) 36

(C) 80

(D) 180 ✓

4. Let f be the function defined by $f(x) = x^3 - \frac{3}{2}x^2 - 6x$. What is the absolute maximum value of f on the interval $[-2, 3]$?

(A) -10

(B) $-\frac{9}{2}$

(C) $\frac{7}{2}$ ✓

(D) 3

5. Let f be the function defined by $f(x) = x\cos x - \sin x$. What is the absolute maximum value of f on the interval $\left[-\frac{\pi}{2}, 2\pi\right]$?

(A) $-\pi$

(B) 2π ✓

(C) 0

(D) 1

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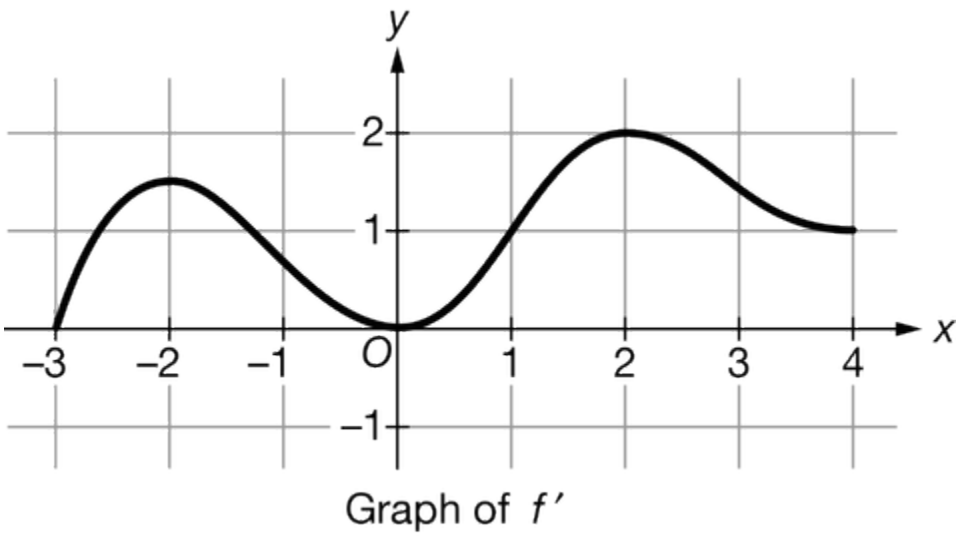
6.



Graph of f'

The graph of f', the derivative of the function f, is shown above. On which of the following open intervals is the graph of f concave down?

(A) $(-2, 0)$ and $(2, 4)$ ✓

(B) $(-3, 2)$ and $(0, 2)$

(C) $(-3, -1)$ only

(D) $(0, 4)$

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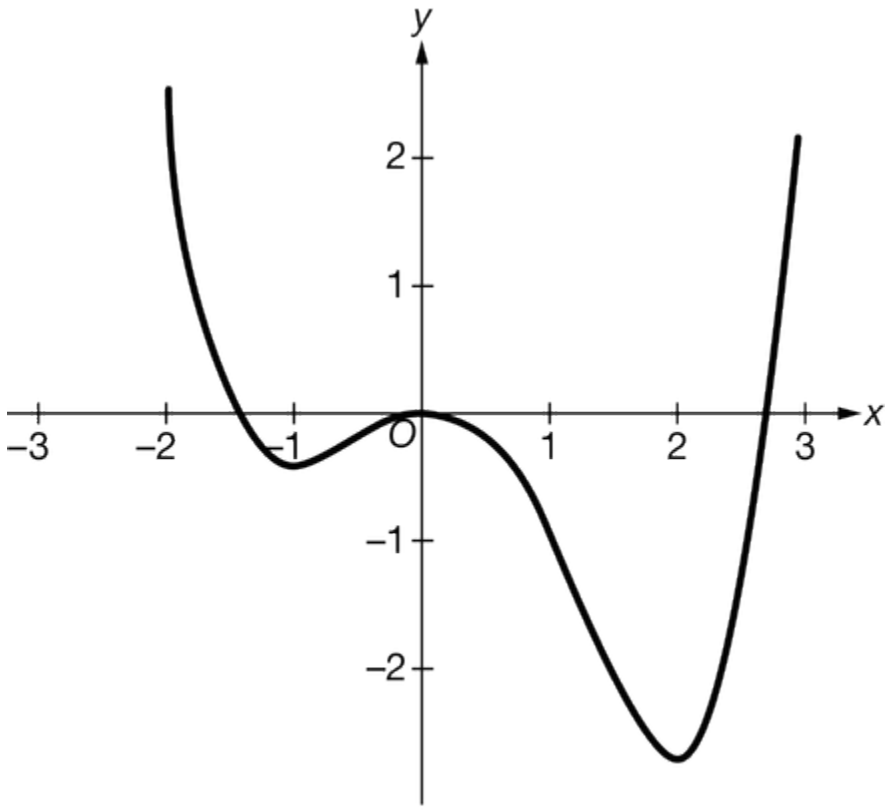
7.



Graph of f'

Let f be the function defined by $f(x) = \frac{x^5}{20} - \frac{x^4}{12} - \frac{x^3}{3}$. The graph of f', the derivative of f, is shown above. On which of the following intervals is the graph of f concave up?

(A) $x < -1$ and $0 < x < 2$

(B) $-1 < x < 0$ and $x > 2$ ✓

(C) $x < \frac{2}{3} - \frac{2\sqrt{10}}{3}$ and $x > \frac{2}{3} + \frac{2\sqrt{10}}{3}$

(D) $\frac{2}{3} - \frac{2\sqrt{10}}{3} < x < \frac{2}{3} + \frac{2\sqrt{10}}{3}$

8. The Second Derivative Test cannot be used to conclude that $x = 1$ is the location of a relative minimum or relative maximum for which of the following functions?

(A) $f(x) = \cos\left(x^2 - 1\right)$, where $f'(x) = -2x \sin\left(x^2 - 1\right)$ and $f''(x) = -2 \sin\left(x^2 - 1\right) - 4x^2 \cos\left(x^2 - 1\right)$

(B) $f(x) = e^{(x-1)^2}$, where $f'(x) = 2(x-1)e^{(x-1)^2}$ and $f''(x) = 4(x-1)^2 e^{(x-1)^2} + 2e^{(x-1)^2}$

(C) $f(x) = \frac{x^3}{3} + x^2 - 3x + 1$, where $f'(x) = x^2 + 2x - 3$ and $f''(x) = 2x + 2$

(D) $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$, where $f'(x) = 4x^3 - 12x^2 + 12x - 4$ and $f''(x) = 12x^2 - 24x + 12$ ✓

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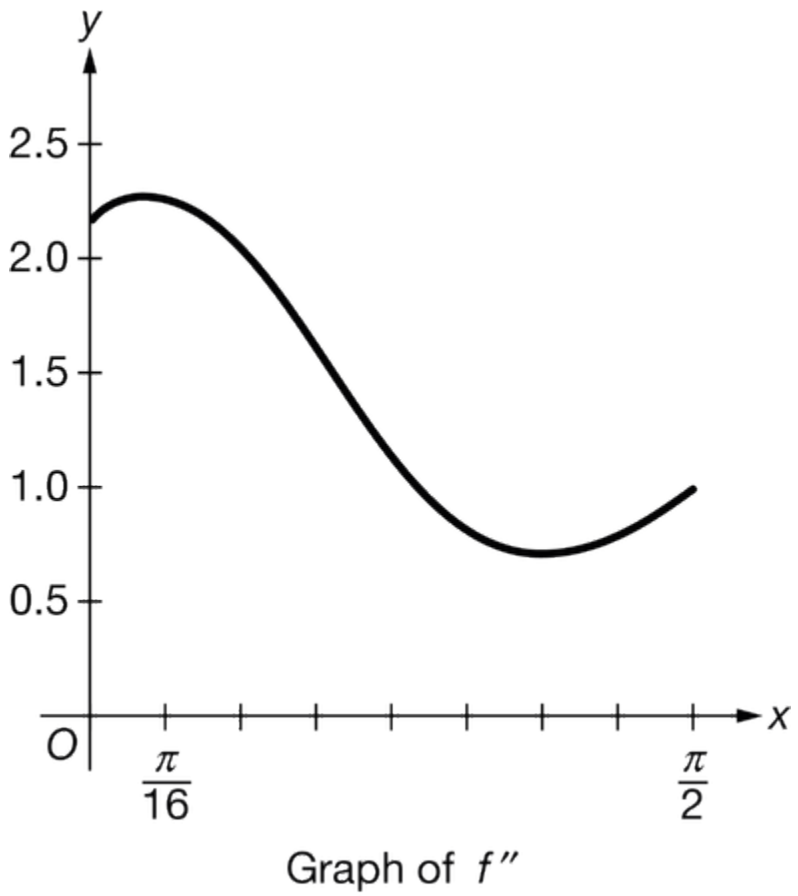
9.



Graph of f''

The graph of f'', the second derivative of the continuous function f, is shown above on the interval $\left[0, \frac{\pi}{2}\right]$. On this interval f has only one critical point, which occurs at $x = \frac{\pi}{16}$. Which of the following statements is true about the function f on the interval $\left[0, \frac{\pi}{2}\right]$?

(A) f has a relative minimum at $x = \frac{\pi}{16}$ but not an absolute minimum.

(B) The absolute minimum of f is at $x = \frac{\pi}{16}$. ✓

(C) f has a relative maximum at $x = \frac{\pi}{16}$ but not an absolute maximum.

(D) The absolute maximum of f is at $x = \frac{\pi}{16}$.

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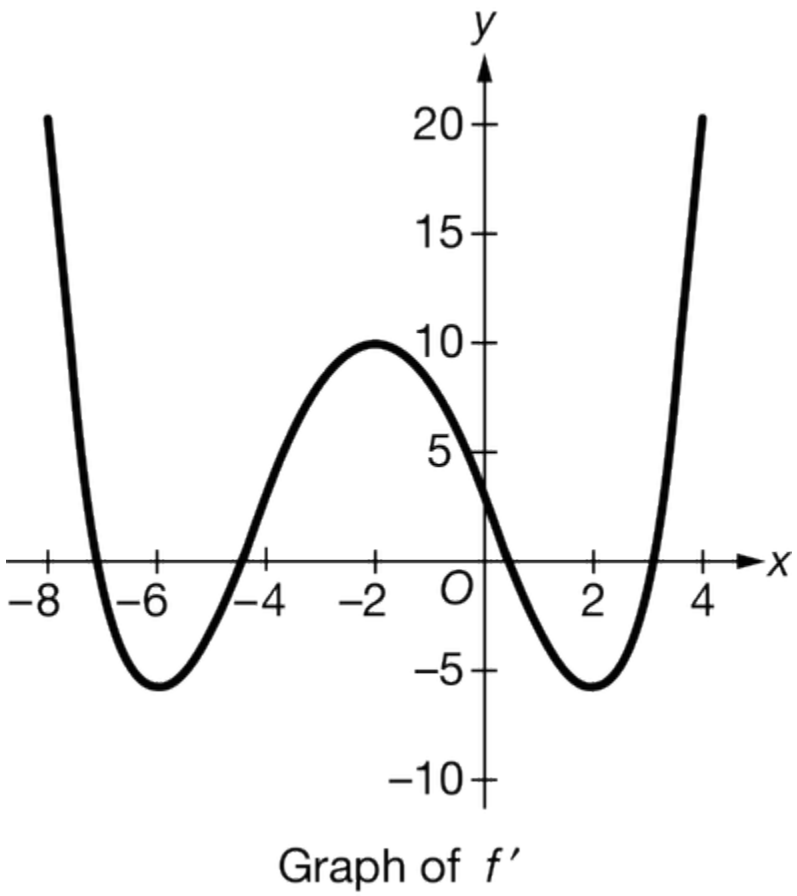
10.



Graph of f'

The graph of f', the derivative of the continuous function f, is shown above on the interval $[-8, 4]$. The graph of f' has horizontal tangent lines at $x = -6$, $x = -2$, and $x = 2$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-8, -6)$

(B) $(-6, -4.5)$ ✓

(C) $(-2, 0)$

(D) $(1, 2)$

11.

x	$0 < x < 1$	$x = 1$	$1 < x < 2$	$x = 2$	$2 < x < 3$	$x = 3$	$3 < x < 4$	$x = 4$	$4 < x < 5$
$f'(x)$	Positive	0	Negative	DNE	Positive	0	Positive	0	Unknown
$f''(x)$	Unknown	-5	Negative	DNE	Negative	0	Positive	0	Unknown

The function f is continuous on the interval $(0, 5)$ and is twice differentiable except at $x = 2$, where the derivatives do not exist (DNE). Information about the first and second derivatives of f for some values of x in the interval $(0, 5)$ is given in the table above. Which of the following statements could be false?

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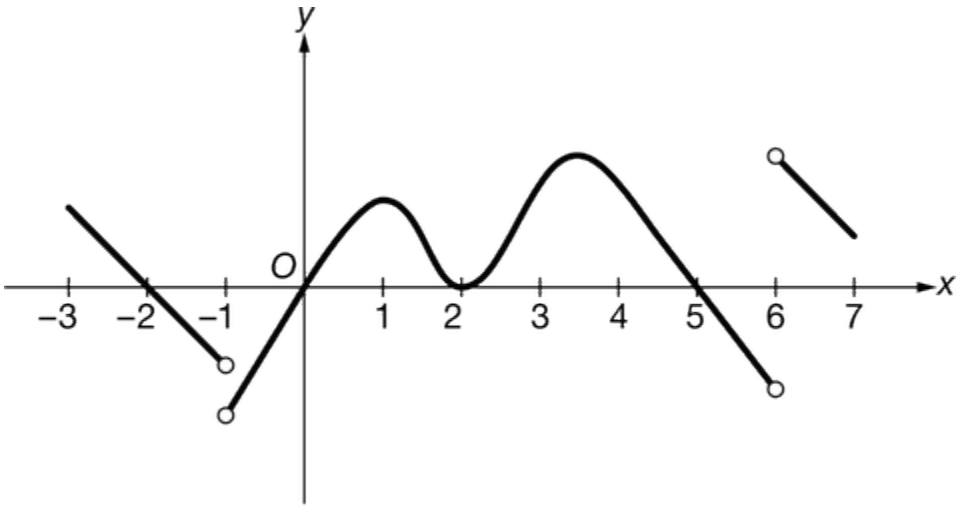
(A) The function f has a relative maximum at $x = 1$.

(B) The function f has a relative minimum at $x = 2$.

(C) The graph of f has a point of inflection at $x = 3$.

(D) The graph of f has a point of inflection at $x = 4$. ✓

12.



Graph of f'

The graph of f', the derivative of the continuous function f, is shown above on the interval $-3 < x < 7$. Which of the following statements is true about f on the interval $-3 < x < 7$?

(A) f has three relative extrema, and the graph of f has one point of inflection.

(B) f has three relative extrema, and the graph of f has four points of inflection.

(C) f has four relative extrema, and the graph of f has two points of inflection.

(D) f has four relative extrema, and the graph of f has four points of inflection. ✓