Taylor and Maclaurin Series Name________________________

1. Find the Taylor series (SHOW THE DERIVATIVES USED) through the $(x-a)^3$ term for: Then approximate tan(.885398)

A. $\tan x$, $a = \frac{\pi}{4}$

y = tanx

y' =

y'(a)

y'' =

y''(a)

y''' =

y'''(a)

tan(.885398) ≈

B. $1 + x^2 + x^3$, $a = 1$

y =

y' =

y'(a)

y'' =

y''(a)

y''' =

y'''(a)

2. Represent $(1-x)^{-2}$ in a Maclaurin series (at least four terms and showing the derivatives used) for $-1 < x < 1$

y =

y' =

y'(a)

y'' =

y''(a)

y''' =

y'''(a)

3. Represent $(1+x)^{0.5}$ in a Maclaurin series (use five terms and show the derivatives used) and use it to approximate $\sqrt{1.1}$

y =

y' =

y'(a)

y'' =

y''(a)

y''' =

y'''(a)

y'''' =

y''''(a)

$\sqrt{1.1} \approx$

Taylor and Maclaurin Series Name

1. Find the Taylor series (SHOW THE DERIVATIVES USED) through the $(x-a)^3$ term for:

A. $\tan x$, $a = \frac{\pi}{4}$

$y = \tan x$

$y' = \sec^2 x$

$y'(a)$ $\sec^2\left(\frac{\pi}{4}\right) = \left(\frac{2}{\sqrt{2}}\right)^2 = 2$

$y'' = 2\sec x \sec x \tan x = 2\sec^2 x \tan x$

$y''(a)$ $2\sec\frac{\pi}{4}\sec\frac{\pi}{4}\tan\frac{\pi}{4} = 2\cdot\frac{2}{\sqrt{2}}\cdot\frac{2}{\sqrt{2}}\cdot 1 = 4$

$y''' = 2\sec^2 x \sec^2 x + \tan x \cdot 2 \cdot 2\sec x \sec x \tan x = 2\sec^4 x + 4\sec^2 x \tan^2 x$

$y'''(a)$ $2\sec^4\left(\frac{\pi}{4}\right) + 4\sec^2\left(\frac{\pi}{4}\right)\tan^2\left(\frac{\pi}{4}\right) = 2(\sqrt{2})^4 + 4(\sqrt{2})^2(1)^2$

$8 + 8 = 16$

$\boxed{\tan x = 1 + 2\left(x-\frac{\pi}{4}\right) + \frac{4}{2!}\left(x-\frac{\pi}{4}\right)^2 + \frac{16}{3!}\left(x-\frac{\pi}{4}\right)^3 + \ldots}$

$\tan(.885398) = 1 + 2(.1) + \frac{4}{2!}(.1)^2 + \frac{16}{3!}(.1)^3 + \ldots$

$x + .1$

real value is

1.223048

B. $1 + x^2 + x^3$, $a = 1$

$y = 1 + x^2 + x^3$

$y' = 2x + 3x^2$

$y'(a)$ $2(1) + 3(1)^2 = 5$

$y'' = 2 + 6x$

$y''(a)$ $2 + 6(1) = 8$

$y''' = 6$

$y'''(a)$ 6

$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$

$\boxed{1 + x^2 + x^3 = 3 + 5(x-1) + \frac{8}{2!}(x-1)^2 + \frac{6}{3!}(x-1)^3 + \ldots}$

2. Represent $(1-x)^{-2}$ in a Maclaurin series (at least four terms and showing the derivatives used) for $-1 < x < 1$

$y = (1-x)^{-2}$ at $x=0$: 1

$y' = -2(1-x)^{-3}(-1) = 2(1-x)^{-3}$

$y'(a)$ @0 : 2

$y'' = -6(1-x)^{-4}(-1) = 6(1-x)^{-4}$

$y''(a)$ @0 : 6

$y''' = -24(1-x)^{-5}\cdot -1 = 24(1-x)^{-5}$

$y'''(a)$ @0 : 24

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$\boxed{(1-x)^{-2} = 1 + 2x + \frac{6}{2!}x^2 + \frac{24}{3!}x^3 + \ldots}$$

$$\frac{1}{(1-x)^2}$$

$(1-.8)^{-2} = 25$

$1 + 2(.8) + \frac{6}{2}(.8)^2 + \frac{24}{6}(.8)^3 =$

3. Represent $(1+x)^{0.5}$ in a Maclaurin series (use five terms and show the derivatives used) and use it to approximate $\sqrt{1.1}$

$y = (1+x)^{1/2}$ @0 : 1

$y' = \frac{1}{2(1+x)^{1/2}} = \frac{1}{2}(1+x)^{-1/2}$

$y'(a)$ @0 : $\frac{1}{2}$

$y'' = -\frac{1}{4}(1+x)^{-3/2}$

$y''(a)$ @0 : $-\frac{1}{4}$

$y''' = \frac{3}{8}(1+x)^{-5/2}$

$y'''(a)$ @0 : $3/8$

$y'''' = \frac{-15}{16}(1+x)^{-7/2}$

$y''''(a)$ @0 : $-15/16$

$\sqrt{1+x}$

$\sqrt{1.1} = (1+.1)$

$= 1 + \frac{1}{2}(.1) - \frac{1}{4\cdot 2!}(.1)^2 + \frac{3}{8\cdot 3!}(.1)^3 - \frac{15}{16\cdot 4!}(.1)$

approx : 1.04880859 4

actual $\sqrt{1.1}$ = 1.0488088 48

$\sqrt{1.1}$

$$(1+x)^{.5} = 1 + \frac{1}{2}x - \frac{\frac{1}{4}}{2!}x^2 + \frac{\frac{3}{8}}{3!}x^3 + \frac{\frac{-15}{16}}{4!}x^4$$

$$\boxed{(1+x)^{.5} = 1 + \frac{1}{2}x - \frac{1}{4\cdot 2!}x^2 + \frac{3}{8\cdot 3!}x^3 - \frac{15}{16\cdot 4!}x^4 + \ldots}$$