Taylor + Maclaurin Series (12.6)

pg 606 - 8 basic power series (BC must memorize)

① Derive a power series for $\cos(x^2)$

WE KNOW $\cos(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \ldots$

$\cos(x^2) =$

SO

$\cos(x^2) = 1 - \frac{1}{2!}(x^2)^2 + \frac{1}{4!}(x^4)^2 -$

$$\boxed{\cos x^2 = 1 - \frac{1}{2!}x^4 + \frac{1}{4!}x^8 - \ldots}$$

② Derive a power series for $\int_0^x t \cos t^5 \, dt$

$\cos t^5 = 1 - \frac{1}{2!}(t^2)^5 + \frac{1}{4!}(t^4)^5 - \ldots$

$= 1 - \frac{1}{2!}t^{10} + \frac{1}{4!}t^{20} - \ldots$

SO $t\cos t^5 = t - \frac{1}{2!}t^{11} + \frac{1}{4!}t^{21} - \ldots$

NOW WE NEED TO $\int$

$$\int_0^x \left(t - \frac{1}{2!}t^{11} + \frac{1}{4!}t^{21} - \ldots\right)dt$$

$$= \frac{t^2}{2} - \frac{1}{12 \cdot 2!}t^{12} + \frac{1}{22 \cdot 4!}t^{22} - \ldots \Big|_0^x$$

$$= \frac{x^2}{2} - \frac{1}{12 \cdot 2!}x^{12} + \frac{1}{22 \cdot 4!}x^{22} - \ldots$$

③ Derive a power series for $\frac{1}{1+x^3}$

SIMILAR TO $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$

let $x = -x^3$

$$\frac{1}{1+x^3} = 1 + (-x^3) + (-x^3)^2 + (-x^3)^3 + (-x^3)^4 + \ldots$$

$$\boxed{\frac{1}{1+x^3} = 1 - x^3 + x^6 - x^9 + x^{12} - \ldots}$$

#5 from yesterday's assignment is a Taylor series expansion of $f(x)$ about $x=1$

$$P(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \ldots$$

Generally, a <u>Taylor series expansion</u> of $f(x)$ about $x=a$ is

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

If $a=0$, the series is called a <u>Maclaurin series</u>.

→ EXPONENT = 5 so S_6

③ Write a 5^{th} degree Taylor polynomial expanded about $x=-1$ if

$f(-1)=7$ $\quad f''(-1) = -.48$ $\quad f^{(4)}(-1) = .36$

$f'(-1)=2$ $\quad f'''(-1)=0$ $\quad f^{(5)}(-1) = -.084$

TAYLOR $f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots$

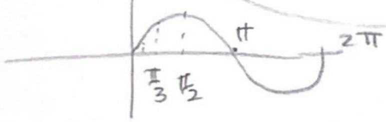
↓ PARTIAL

$$P(x) = 7 + 2(x+1) + \frac{(-.48)}{2!}(x+1)^2 + 0 + \frac{.36}{4!}(x+1)^4 + \frac{(-.084)}{5!}(x+1)^5$$

$$\boxed{P(x) = 7+2(x+1) - .24(x+1)^2 + .015(x+1)^4 - .0007(x+1)^5}$$

④ Write a Taylor series for $f(x) = \sin x$ about $x = \pi/3$

CHANGE OF CENTER



$f(\frac{\pi}{3}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$f'(\frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f''(\frac{\pi}{3}) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$$\boxed{f(x) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x-\frac{\pi}{3}\right) + \frac{-\frac{\sqrt{3}}{2}}{2!}\left(x-\frac{\pi}{3}\right)^2 + \ldots}$$