Tower Project Paper

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Section 9B

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Part One

It all began when a millionaire decided that she wanted a geometrically-astute tower to be constructed in her 42’ x 42’ plot of land. She specifically inquired for the tower to be built by hand with all the calculations done manually. Sean Taylor and Gio Mandwee took up this challenge.

She requested a log cabin with a regular dodecagonal base to be built in her secluded plot of land. On the interior of this cabin, she requested a floor that made her feel like she is walking on water every time she looks down.

The two builders of this project were given many constraints and standards to meet for their construction. A few of these constraints include not being able to use three feet of each side of the plot, having an aquarium built into the flooring, and making the walls one foot thick.

Part Two

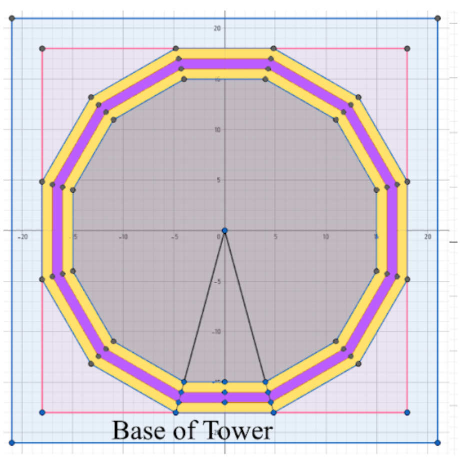

Figure 1. Base of the Tower in the Plot

Figure one shows the base of the tower in the plot of land. The base consists of four regular dodecagons that will be used to determine the location of certain constructional elements.

Central angle = 360/n
[redacted]

Figure 2. Steps to Finding Central Angle of Dodecagon

Figure 2 above shows the process of finding the central angle measures of any given regular polygon where n corresponds to the number of sides. In the case of a dodecagon, n equals 12, leading to the deduction that the central angle measurement of a dodecagon is 30 degrees.

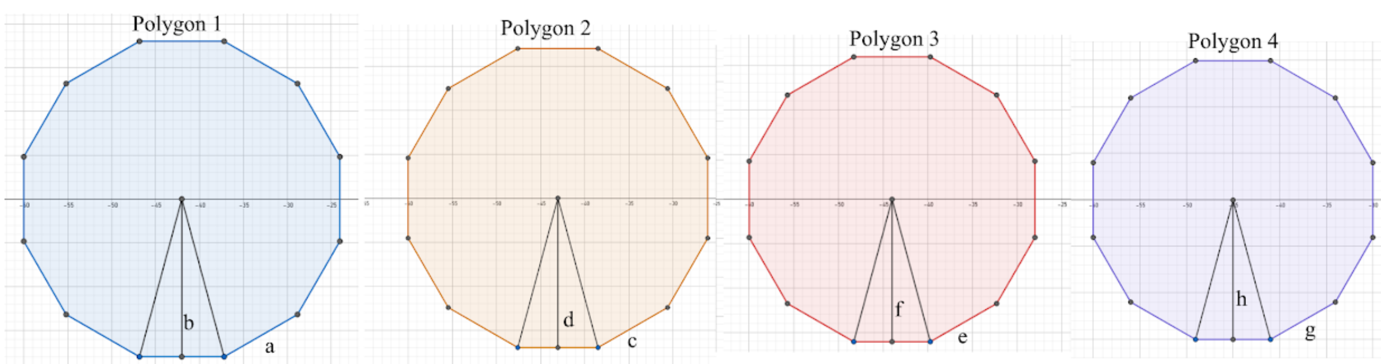

Figure 3. Diagrams of Polygons One Through Four

Polygon 1	$36tan(15) \approx$
Polygon 2	$34tan(15) \approx$
Polygon 3	$32tan(15) \approx$
Polygon 4	$30tan(15) \approx$

Figure 4. Side Length of Polygons

For a regular polygon with a number of sides divisible by four, one side is equal to $2tan(1/2c)a$ where c is equal to the measure of a central angle, and a is equal to the measure of the apothem. For polygon one, a equals 18 and c equals 30. This simplifies to 36tan(15). For polygon two, a equals 17 and c equals 30. This simplifies to 34tan(15). For polygon three, a equals 16 and c equals 30. This simplifies to 32tan(15). For polygon four, a equals 15 and c equals 30. This simplifies to 30tan(15).

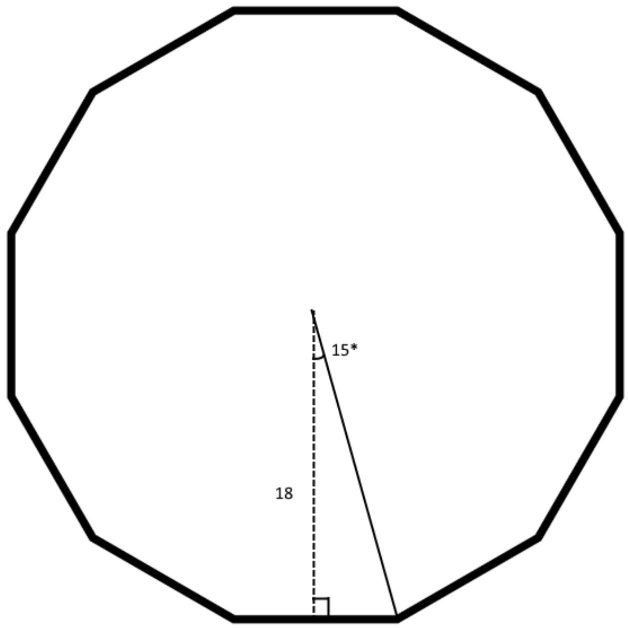

Figure 5. Polygon One

Polygon 1	18
Polygon 2	17
Polygon 3	16
Polygon 4	15

Figure 6. Apothem of Polygons

Since the plot size is 42’ x 42’, only 36’ x 36’ of it is permitted for use, and the sides are centered, the apothem of polygon one is equal to 18’ since 18’ is half of the total 36’ needed to form one side of the plot (reference figure 5). Since each polygon goes inward 1’, each apothem gets 1’ smaller for every polygon past polygon one. A formula for this while $x \geq 1$ is $a = 19 - x$, where a is equal to the apothem length and x equals the polygon number.

Polygon 1	[redacted]
Polygon 2	[redacted]
Polygon 3	[redacted]
Polygon 4	[redacted]

Figure 7. Area of Polygons

Using the formula (nsa)/2, where n is the number of sides of a regular polygon, s is one side length, and x is the length of the apothem, the area of that polygon is given. By substituting n for 12, since all of these polygons have 12 sides, and s for 2xtan(15), where a is the length of the apothem, the formula of $12x^{2}tan(15)$ is given to find the area of a regular dodecagon.

Part Three

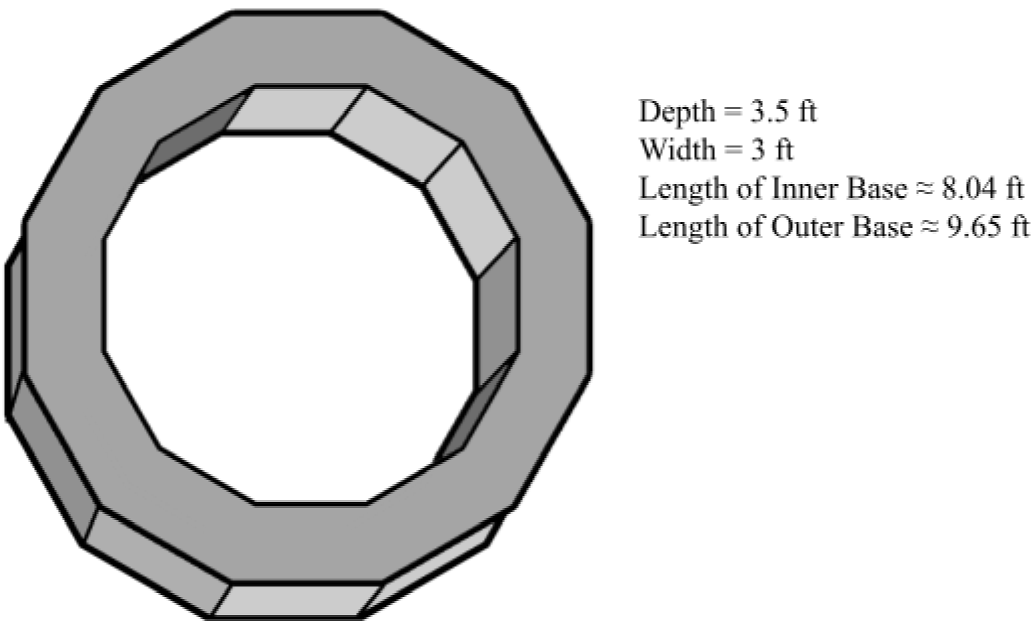

Figure 8. Dimensions of Footing

Figure 8 shows the dimension used to construct the footing of the tower. The depth had to be 3.5 ft, and it had to extend from polygon one to polygon four making it 3 ft wide. The length of the inner base is equal to the side length of polygon four, which is approximately 8.04 ft. The length of the outer base is the same concept, but except for the side length of polygon 4 being the base, it is the side length of polygon 1, which is approximately 9.65'.

Volume of Footing =	(Area of polygon 1 - Area of polygon 4) * depth
Volume of Footing =	[redacted]
Volume of Footing ≈	

Figure 9. Volume of Footing

Figure 9 demonstrates the process of finding the volume of the footing given the area of polygon one, the area of polygon four, and the depth of the footing.

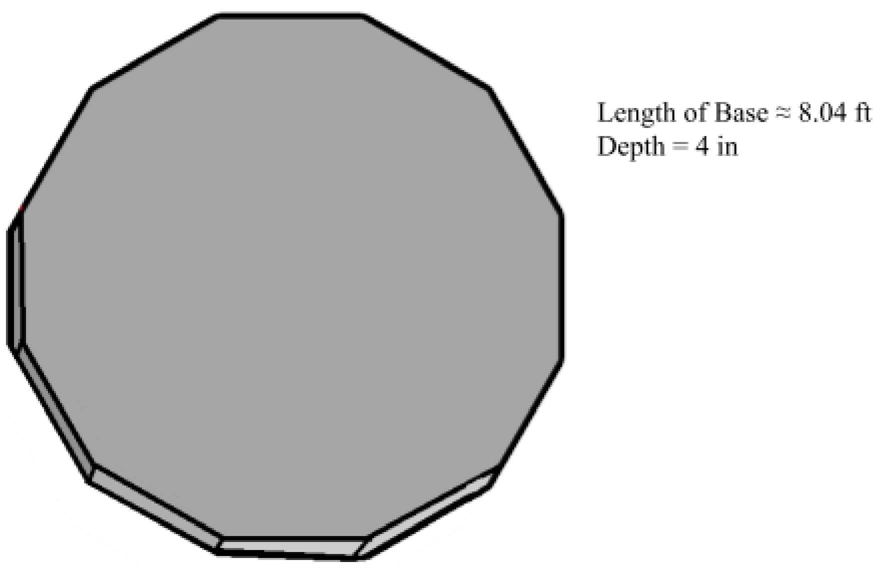

Figure 10. Dimensions of Flooring

Figure 10 illustrates the floor along with the dimensions used to construct it. The floor had to extend to polygon four, making the side length of it equal to a side length of polygon four (≈ 8.04 ft). The depth of the flooring had to be 4 in.

Volume of Floor =	(Area of polygon four) * depth
Volume of Floor =	[redacted]
Volume of Floor ≈	

Figure 11. Volume of Plexiglass Needed for Floor

Figure 11 demonstrates the process of finding the volume of the flooring/plexiglass given the area of polygon four and the depth of the flooring. Since the area of polygon four is in feet and the depth of the flooring is in inches, the depth had to be converted from 4 inches to ⅓ of a foot. The area of polygon four was used as the surface area of the flooring since the flooring is made along the base of polygon four, thus making their areas equal.

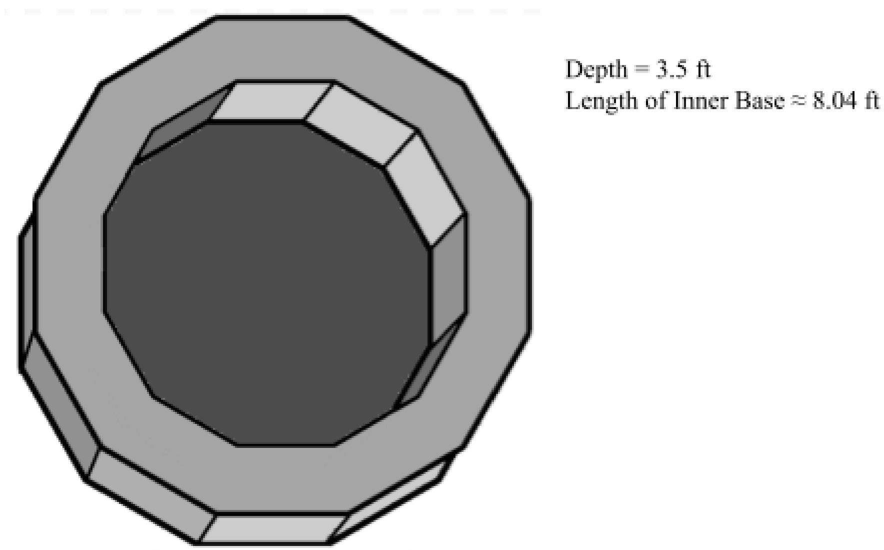

Figure 12. Dimensions of Aquarium

Figure 12 shows the dimensions required for the construction of the aquarium. Since the aquarium is located inside the footing of polygon four, it must be 3.5 feet deep and have a side length of approximately 8.04 ft.

Volume of Water =	(Area of polygon four) * depth * fillPercentage
Volume of Water =	[redacted]
Volume of Water ≈	

Figure 13. Volume of Plexiglass Needed for Floor

Figure 13 demonstrates the process of finding the volume of the aquarium given the area of polygon four and the depth of the aquarium. The volume of water that can fit in the aquarium can be determined by multiplying the area of polygon four by the depth of the aquarium by 0.75 to account for filling the tank with 75% water.

Cost of Concrete =	$V_{Flooring}$ / 27
Cost of Concrete =	[redacted]
Cost of Concrete ≈	

Figure 14. Cost of Concrete

Figure 14 shows the process of finding the cost of concrete. The concrete is sold for $115 per cubic yard. To find the amount of cubic yards, the number of cubic feet must be divided by 27 to be converted to cubic yards (41.26). From here, the number needs to be rounded up since it is not possible to buy a fraction of a bag of concrete (42). This number can be multiplied by 115 to give the final cost of concrete in dollars ($4830).

Cost of Plexiglass =	(A of P_4/4608) * 1100
Cost of Plexiglass =	[redacted]
Cost of Plexiglass ≈	

Figure 15. Cost of Plexiglass

Figure 15 shows the process of finding the cost of plexiglass. The plexiglass is sold for $1100 per 48” x 96” x 4” sheet. Since the sheet of plexiglass is the same height as the plexiglass sheet used in the flooring, the height of 4” can be disregarded on both measurements, thus leaving us with a 48” x 94” plexiglass sheet to check how much it fits into the area of the base of the regular dodecagon. After 2700tan(15) is simplified, that results in the square feet that comprises the flooring. This result needs to be multiplied by 144 to give the number of square inches. This result needs to be divided by 4608, giving the number of plexiglass sheets required (22.608). Since it is not possible to buy .608 of a plexiglass sheet, this needs to be rounded up to 23 plexiglass sheets, resulting in $.

Part Four

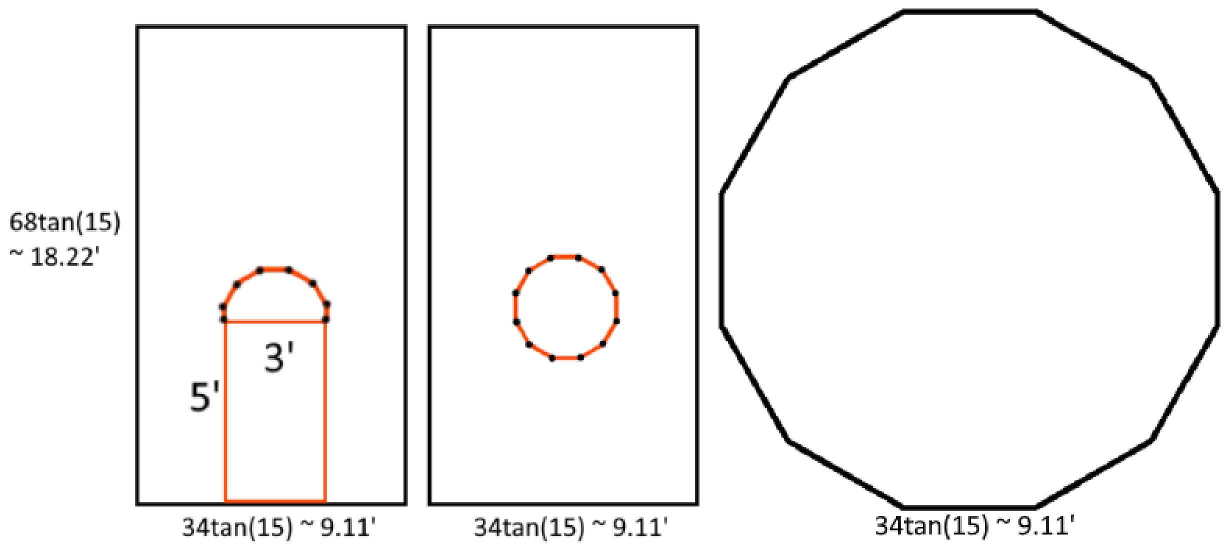

Figure 16. Dimensions of Door, Wall, Window, and Outer Wall base

The base and height of the door were requested to be 3 ft and 5 ft respectively. The apothem of the polygon above the rectangular section of the door measures 1.5 ft. The way this number was calculated was since the base of the door is 3 ft and the top of the door is a perpendicular bisector to two sides of the polygon, the apothem must be half of 3 ft.

Area of Door Panel =	A_{wall} - A_{door}
Area of Door Panel =	(Base * Height) - (Base * Height + (nsa/4))
Area of Door Panel =	[redacted]
Area of Door Panel =	[redacted]
Area of Door Panel ≈	[redacted]

Figure 17. Area of Door Panel

Figure 17 demonstrates the process of finding the area of the panel housing the door by finding the area of the wall panel and subtracting the area of the door. Since the door panel is half of a regular polygon divisible by four, the formula $2tan(1/2c)a$ can be used to find the side length given that c is the measure of one central angle of that shape and a is the measure of the apothem of the shape. This formula equates in 3tan(15) for the length of one side of the full shape. Now that the number of sides, side length, and apothem length are known, the formula nsa/4 can be used to calculate half the area of the shape. This results in approximately sqft for the area of the window panel.

Area of Window Panel =	A_{wall} - A_{window}
Area of Window Panel =	(Base * Height) - (nsa/2)
Area of Window Panel =	[redacted]
Area of Window Panel =	[redacted]
Area of Window Panel ≈	[redacted]

Figure 17. Area of Window Panel

Figure 17 demonstrates the process of finding the area of the panel housing the window by finding the area of the wall panel and subtracting the area of the window. Since the window panel is a regular polygon divisible by four, the formula $2tan(1/2c)a$ can be used to find the side length given that c is the measure of one central angle of that shape and a is the measure of the apothem of the shape. This formula equates in 3tan(15). Now that the number of sides, side length, and apothem length are known, the formula nsa/2 can be used to calculate the area of the shape. This results in approximately sqft for the area of the window panel.

Area of Wall Panel =	Base * Height
Area of Wall Panel =	[redacted]
Area of Wall Panel ≈	[redacted]

Figure 18. Area of Wall Panel

Figure 18 demonstrates the process of finding the area of a panel that is not housing any additional components. Since the base is 34tan(15) and the height is 68tan(15), multiplying the base and height together result in the area of the panel, which is approximately 165.99 sqft.

LSA of Outer Prism =	$2*A_{windowPanel} + 1*A_{doorPanel} + 9*A_{wallPanel}$
LSA of Outer Prism =	[redacted]
LSA of Outer Prism =	[redacted]
LSA of Outer Prism ≈	[redacted]

Figure 19. Lateral Surface Area of Outer Prism

Figure 19 depicts the process used to find the lateral surface area of the outer prism part of the tower. Since 2 window panels, 1 door panel, and 9 wall panels are used in the construction of the outer prism of the tower, adding together the areas of all these panels results in the total lateral surface area of the outer prism (sqft).

Part Five

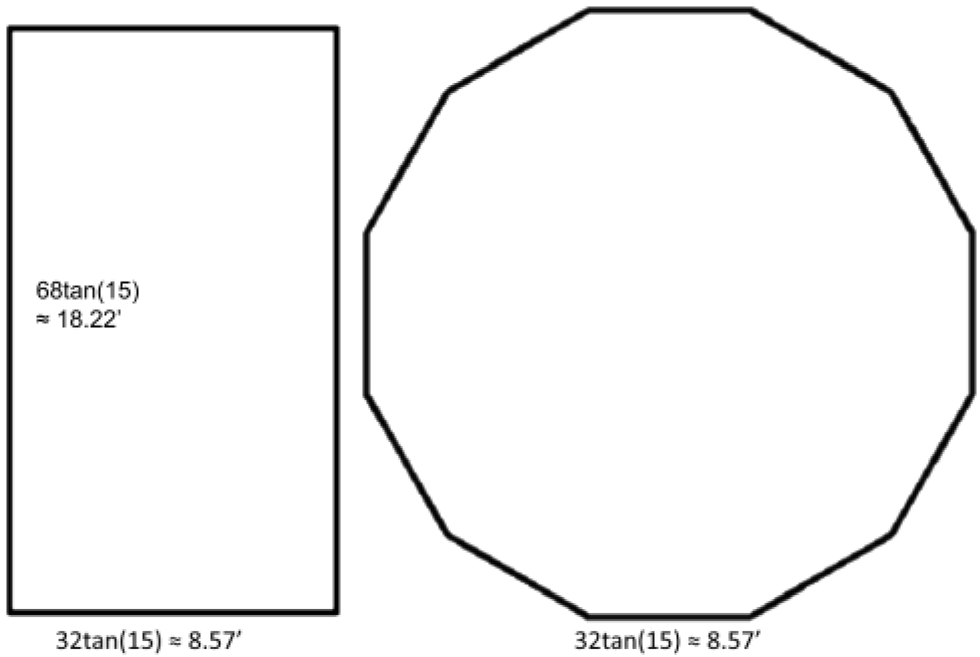

Figure 20. Dimensions of Inner Prism

Figure 20 shows the dimensions used to construct the inner walls. Since the inner walls rest on polygon three, the base of the inner wall is approximately 8.57 ft. The height of the wall is approximately 18.22 ft since it is the same height as the outer wall.

Volume of Inner Prism =	(Area of polygon tower) * height
Volume of Inner Prism=	[redacted]
Volume of Inner Prism=	[redacted]
Volume of Inner Prism ≈	[redacted]

Figure 21. Volume of Inner Prism

Figure 21 shows the process of finding the volume of the inner prism of the tower. Since it can be viewed as a dodecagon prism with a base with the area of polygon 3 and a height of 68tan(15). Since the volume of a prism is calculated by multiplying the area of the base by the height of the prism, the area of polygon 3 is multiplied by 68tan(15) to produce a result of approximately cubic ft.

Part Six

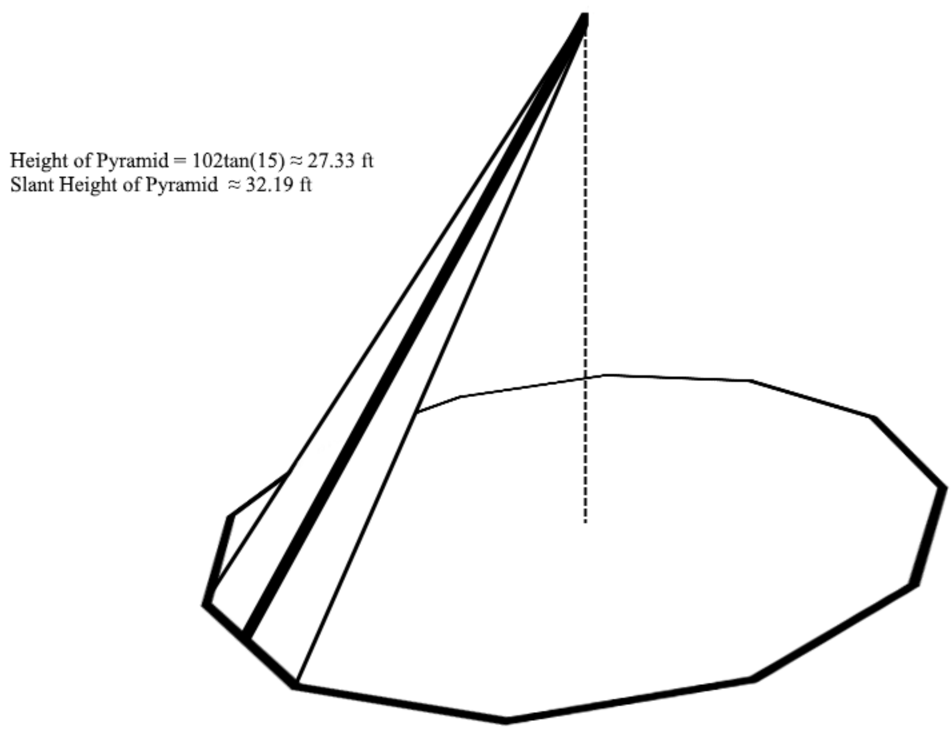

Figure 22. Dimensions of Outer Pyramid

Figure 22 shows the dimensions used to construct the outer pyramid of the roof. The height was approximately 27.33 ft and the slant height was 19.69 ft. The height was calculated by multiplying a side length of polygon 2 (34tan(15)) by three to get 102tan(15) which is approximately 27.33 ft. The way the slant height was calculated will be explained later.

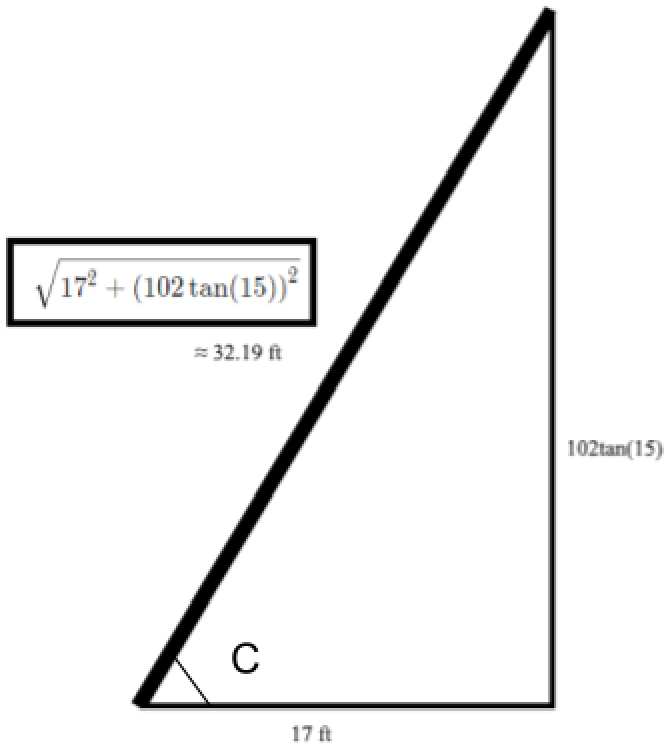

Figure 23. Visualization of Slant Height of Outer Pyramid

Slant Height of Outer Pyramid Squared =	$apothem^2 + height^2$
Slant Height of Outer Pyramid Squared =	[redacted]
Slant Height of Outer Pyramid =	[redacted]
Slant Height of Outer Pyramid ≈	[redacted]

Figure 24. Slant Height of Outer Pyramid

Figure 24 demonstrates the process used to calculate the slant height of the outer pyramid. The triangle cross section represented in figure 23 was seen and due to the pythagorean theorem, the base of the triangle (apothem of pyramid base) squared plus the height squared must equal the slant height squared. After this result is computed, both sides are square rooted to result in the slant height of approximtaely 32.19 ft.

tan(θ) =	opposite/adjacent
tan($\angle$C) =	[redacted]
$\angle$C =	[redacted]
$\angle$C ≈	[redacted]

Figure 25. Angle Between Prism Base and Pyramid Face

Figure 25 demonstrates the process used to calculate the angle measure formed between the prism base and the pyramid face found at the foot of the slant height, which is $\angle$C. First, it is identified that the tangent is equal to the side opposite of angle theta over the side adjacent to it. Next, theta is substituted for $\angle$C, opposite is substituted for 102tan(15), and adjacent is substituted for 17. In the next step, 102tan(15)/17 is simplified to 6tan(15), and the inverse tangent of both sides was taken to isolate $\angle$C. A calculator was used to approximate $\angle$C to be approximately .

Part Seven

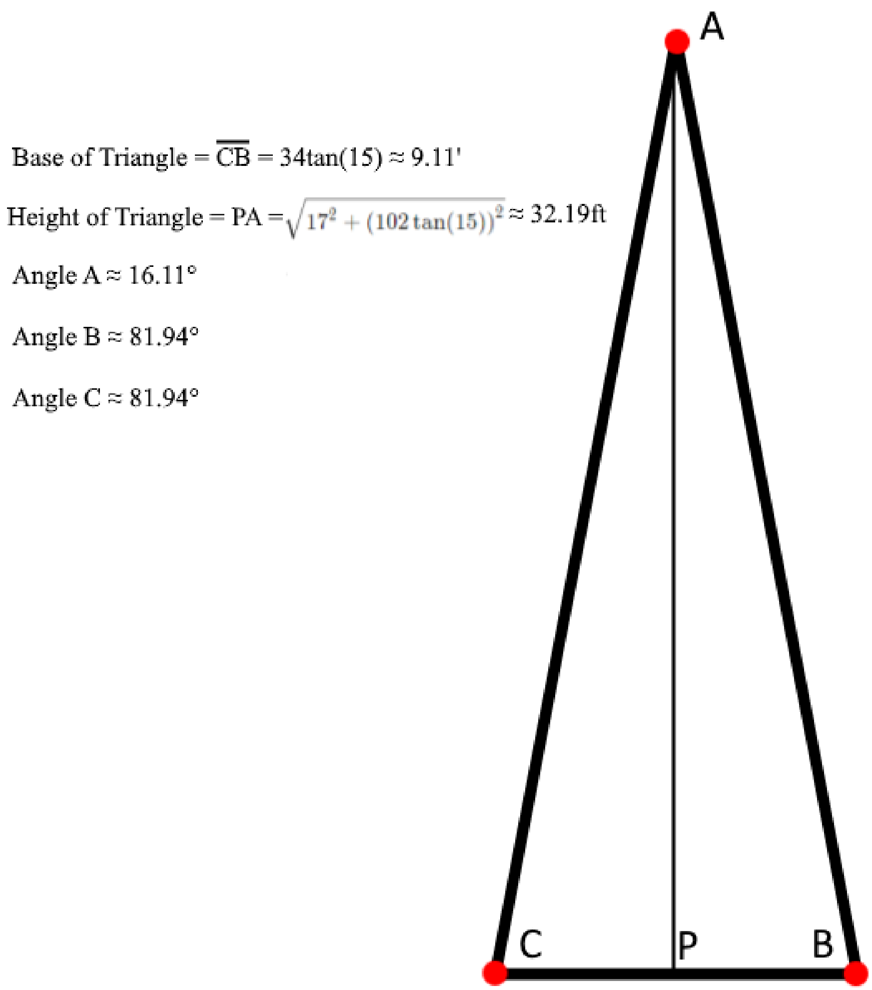

Figure 26. Dimensions of One Lateral Face of Outer Pyramid

Figure 26 shows the dimensions that were used to construct one lateral face of the outer pyramid. Since the pyramid extends to polygon 2, the length of the base is equal to one side of polygon 2, which is 34tan(15), which is approximately 9.11 ft. The height of the triangle is equal to the slant height of the pyramid in part 6, meaning that the height of the triangle is approximately 32.19 ft. The way that the angle measures of angle A, B, and C were calculated will be explained later in the paper.

∠CAB =	2∠CAP
∠CAP =	[redacted]
∠CAB =	[redacted]
∠CAB =	[redacted]
∠CAB ≈	[redacted]

Figure 27. Measure of Angle A

Figure 27 demonstrates the process used to find the measure of ∠CAB. In figure 26, since ΔABC is an isosceles triangle, it can be identified that ∠CAP is half the measure of ∠CAB. Since ΔCPA forms a right triangle, right triangle trigonometry can be used to determine that tan(∠CAP) = 17tan(15)/√17^2 + (102tan(15))^2. After taking the arctangent of both sides, it can be determined that ∠CAP = arctan(17tan(15)/√17^2 + (102tan(15))^2). The 2 can be distributed into the numerator, then the fraction can be approximated to equal approximately 16.11, meaning that ∠CAB is approximately 16.11 degrees.

tan(∠B) =	√17^2 + (102tan(15))^2/17tan(15)
∠B =	[redacted]
∠B ≈	[redacted]
∠C ≈	[redacted]

Figure 28. Measures of Angles B and C

Figure 28 demonstrates the process used to find the measure of ∠B and ∠C. Since ΔABC is an isosceles triangle, ∠B equals ∠C. Since the tangent of a given right triangle (ΔCPA in this case) is equal to the opposite side over the adjacent side, the tangent of ∠B would equal √17^2 + (102tan(15))^2/17tan(15). The arctangent of both sides can be taken and results in the equation ∠B = arctan(√17^2 + (102tan(15))^2/17tan(15)). This approximates to 81.94, meaning that ∠B is approximately 81.94 degrees. Since ∠C and ∠B are congruent, ∠C also is approximately degrees.

A =	1/2bh
A =	[redacted]
A =	[redacted]
A ≈	[redacted]

Figure 29. Area of One Lateral Face of Outer Pyramid

Figure 28 demonstrates the process used to find the area of one lateral face of the outer pyramid. Since the area of any triangle is calculated by 1/2bh, the area of one lateral face of the outer pyramid is ½ * 34tan(15) * √17^2 + (102tan(15))^2. ½ of 34tan(15) can be simplified to 17tan(15). 17tan(15) + (102tan(15))^2 is approximately sqft.

LSA of Outer Pyramid =	$12 * A_{LateralFace}$
LSA of Outer Pyramid =	[redacted]
LSA of Outer Pyramid =	[redacted]
LSAof Outer Pyramid ≈	[redacted]

Figure 29. Lateral Surface Area of Outer Pyramid

Figure 28 demonstrates the process used to total lateral surface area of the outer pyramid. Since the lateral surface area is comprised of 12 of the triangles calculated in figure 27, the lateral surface area can be calculated by multiplying 17tan(15) * √17^2 + (102tan(15))^2 by 12 to get a result of approximately sqft.

Part Eight

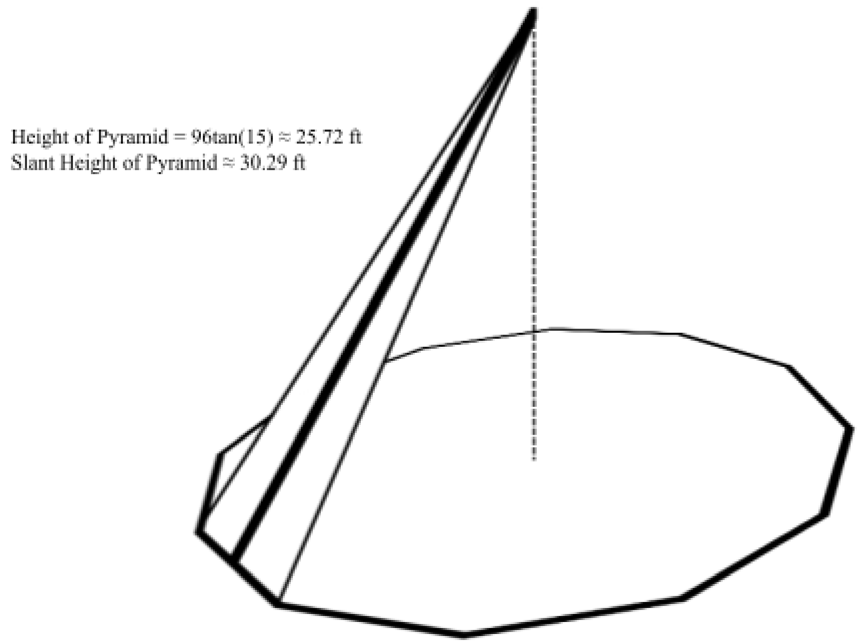

Figure 30. Dimensions of Outer Pyramid

Figure 30 shows the dimensions used to construct the inner pyramid of the roof. The height was approximately 25.72 ft and the slant height was approximately 30.29 ft. The height was calculated by multiplying a side length of polygon 3 (32tan(15)) by three to get 96tan(15) which is approximately 25.72 ft. The way the slant height was calculated will be explained later.

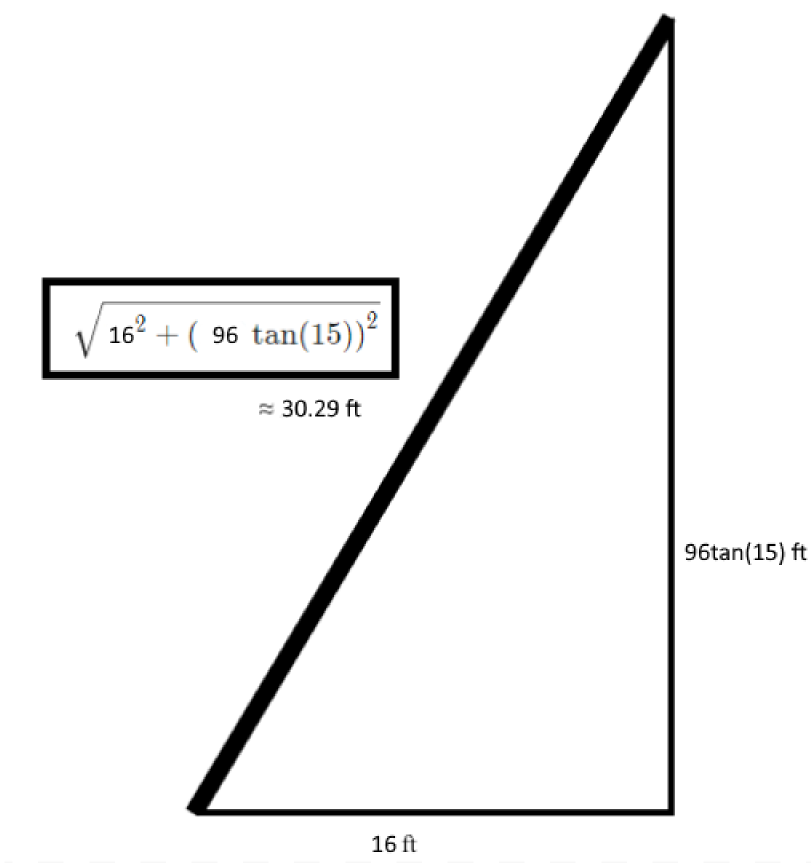

Figure 31. Visualization of Slant Height of Inner Pyramid

Slant Height of Inner Pyramid Squared =	$apothem^2 + height^2$
Slant Height of Inner Pyramid Squared =	[redacted]
Slant Height of Inner Pyramid =	
Slant Height of Inner Pyramid ≈	

Figure 32. Slant Height of Inner Pyramid

Figure 32 demonstrates the process used to calculate the slant height of the inner pyramid. The triangle cross section represented in figure 31 was seen, and due to the pythagorean theorem, the base of the triangle (apothem of pyramid base) squared plus the height squared must equal the slant height squared. After this result is computed, both sides are square rooted to result in the slant height of approximtaely 30.29 ft.

Volume of Inner Pyramid =	⅓ * A_{base} * Height
Volume of Inner Pyramid =	[redacted]
Volume of Inner Pyramid =	[redacted]
Volume of Inner Pyramid ≈	[redacted]

Figure 33. Volume of Inner Pyramid

Figure 33 demonstrates the process used to calculate the volume of the inner pyramid. Since the formula for the volume of any pyramid with a regular polygon base is ⅓ * A_{base} * height, the area of this pyramid's base and its height can be substituted and evaluated to give the volume of the pyramid. In figure 7, the area of polygon 3, which is the same as the area of the pyramid base, was calculated to equal 3072tan(15). In figure 30, the height of the inner pyramid was calculated to be 96tan(15). Now that the values for the area of the pyramid's base and the height of the pyramid are known, they can be substituted into the formula to result in a volume of $98304tan^{2}(15)$, which is approximately cubic feet.

Part Nine

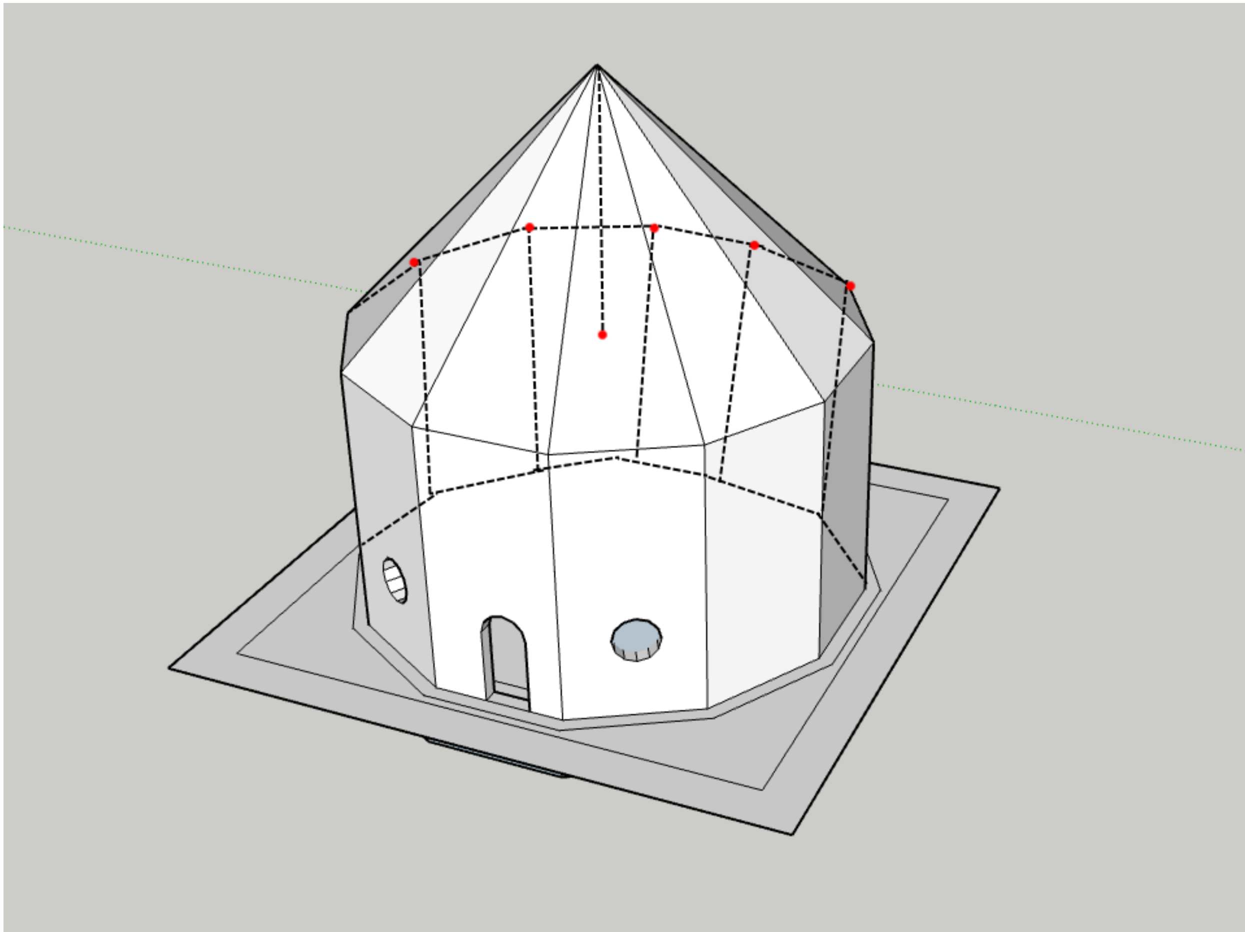

Figure 34. Completed Tower

Figure 34 shows how the tower will look when all parts of the tower are constructed.

LSA of Outer Tower =	$LSA_{Outer\ Prism} + LSA_{Outer\ Pyramid}$
LSA of Outer Tower ≈	[redacted]
LSA of Outer Tower ≈	

Figure 35. Lateral Surface Area of Outer Tower

Figure 35 demonstrates the process of finding the lateral surface area of the outer tower. Since the lateral surface area of the outer tower comprises the lateral surface area of the outer pyramid (found in figure 29) and outer prism (found in figure 19), the total lateral surface area can be found by adding these two values together to get a result of approximately square feet.

LSA of Outer Tower =	$V_{\text{Inner Prism}} + V_{\text{Inner Pyramid}}$
LSA of Outer Tower ≈	[redacted]
LSA of Outer Tower ≈	[redacted]

Figure 36. Volume of Inner Tower

Figure 36 demonstrates the process of finding the volume of the inner tower. Since the inner tower is composed of the inner prism and inner pyramid, the volumes of the inner prism (found in figure) and inner pyramid (found in figure) can be added together to result in a total volume of approximately cubic feet.

Part 10

A plentiful quantity of log cabins have been constructed by wealthy individuals in the past, but none come close to the sheer beauty and space provided by this wonderfully-constructed cabin. The millionaire was at first hesitant as to whether or not she would have enough space to decorate the interior of her cabin, but with 22055.97 cubic ft of inner volume to work with, she is beyond satisfied with her cabin. Despite her being a wealthy woman, this cabin was objectively very pricey to construct. The labor, mathematical prowess, and materials required to construct this building inquire a very steep price for its construction, but since the builders are the best of the best and produced the best possible log cabin, this price is justified.

The project went very smoothly in terms of calculations. The largest issue that occurred was accidentally having the calculator in radians mode which was immediately noticed and fixed. The construction of the scale model, however, was noticed to be much too large at a 1’ to ½” scale, and so was downscaled to a 1’ to ¼” scale before the construction of the model began.