Answers to Worksheet on Lagrange Error Bound

1. (a) $6+8(x-5)+15(x-5)^2+8(x-5)^3$

   (b) $f(5.2)\approx P_3(5.2)=8.264$

   $|R_3(5.2)|\le 0.005$

   (c) $8.259\le f(5.2)\le 8.269$

   (d) No, $f(5.2)$ can't equal 8.254 because 8.254 does not lie in the interval found in part (c).

2. (a) $\frac{\sqrt{3}}{2}-x-\frac{2\sqrt{3}}{2!}x^2+\frac{4}{3!}x^3$

   (b) $\left|R_3\left(\frac{1}{10}\right)\right|\le\left|\frac{16\left(\frac{1}{10}\right)^4}{4!}\right|=\frac{2^4\left(\frac{1}{2^4\cdot 5^4}\right)}{4!}=\frac{1}{5^4\cdot 4!}=\frac{1}{625\cdot 24}=\frac{1}{15{,}000}<\frac{1}{12{,}000}$

3. (a) $1+\frac{x-3}{2}-\frac{(x-3)^2}{4\cdot 2!}+\frac{3(x-3)^2}{8\cdot 3!}$

   (b) 1.310

   (c) Since $f^{(4)}(x)$ is increasing on [3, 4], $f^{(4)}(x)<6$ on [3, 3.7] so

   $|\text{Error}|<\left|\frac{6(3.7-3)^4}{4!}\right|=0.060<0.08$.

   (d) Yes, $1.250\le f(3.7)\le 1.370$ so $f(3.7)$ could equal 1.283.

4. The series has terms that are alternating in sign, decreasing in magnitude, and having a limit of 0 so the error is less than the absolute value of the first truncated term by the Alternating Series Remainder.

   $|\text{Error}|<|\text{6th term}|$ so $|\text{Error}|<\frac{5}{6!}$ or 0.012.

5. (a) $P(x)=\frac{\sqrt{3}}{2}-\frac{3x}{2}-\frac{9\sqrt{3}x^2}{2\cdot 2!}+\frac{27x^3}{2\cdot 3!}+\frac{81\sqrt{3}x^4}{2\cdot 4!}$

   (b) $|R_4(x)|=\left|\frac{f^{(5)}(z)(x-0)^5}{5!}\right|\le\left|\frac{243x^5}{5!}\right|$ so $\left|R_4\left(\frac{1}{6}\right)\right|\le\left(\frac{243}{5!}\right)\cdot\left(\frac{1}{6}\right)^5=\frac{1}{5!2^5}=\frac{1}{(120)(32)}<\frac{1}{3000}$

6. The series has terms that are alternating in sign, decreasing in magnitude, and having a limit of 0 so the error is less than the first truncated term by the Alternating Series Remainder.

   $\int_0^1 e^{-x^2}\,dx\approx 1-\frac{1}{3}+\frac{1}{10}-\frac{1}{42}=\frac{43}{105}$. $|\text{Error}|<\frac{1}{216}<\frac{1}{200}$.

7. 0.003